1. (a) $L_0 = 1\,\text{m}$ : $L = L_0 / \gamma = 0.75\,\text{m}$ : so $\gamma = 4/3$

$\gamma = (1 - u^2/c^2)^{-1/2}$ : so $(1 - u^2/c^2) = 9/16$

so $u^2/c^2 = 7/16$ : and $u = \sqrt{7}c/4$ $\underline{= 0.66c\ (2\times10^8\ \text{ms}^{-1})}$.

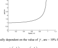
(b) $t_T = \gamma t_E$ :

so $100s = \gamma \times 95s$ and $\gamma = 100/95$

$\Rightarrow (1 - u^2/c^2) = (95/100)^2$ and $\underline{u = 0.3c\ (0.9\times10^8\ \text{ms}^{-1})}$.

(c)

$\gamma$



$u/c$

i.e. relativistic effects, typically dependent on the value of $\gamma$, are ~ 10% for $u/c \sim 0.4$

(d) $\gamma = (1 - u^2/c^2)^{-1/2} = 1 + \frac{1}{2}\left(\frac{u^2}{c^2}\right) + \cdots O\left(\frac{u^4}{c^4}\right)$.

2. Distance to alpha Centauri is 4.4 light years (as measured in the earth frame of reference). Astronauts observe this distance Lorentz contracted as $(4.4/\gamma)$ light years and so travelling at speed $u$ the journey will take $4.4\ \text{light years}/\gamma u = 10\ \text{years}$.

i.e. $\gamma u = 4.4c/10 \rightarrow \dfrac{u/c}{\sqrt{1 - u^2/c^2}} = 0.44$

So $u^2/c^2 = (0.44)^2(1 - u^2/c^2) \rightarrow u^2/c^2 = (0.44)^2/\left(1 + (0.44)^2\right)$

And $u = 0.40c$ $\underline{= 1.2\times10^8\ \text{ms}^{-1}}$

According to Earth clocks: $t = \text{distance}/u$

i.e. $t_E = 4.4\ \text{light years}/0.4c$ $\underline{= 11\ \text{years}}$.

Same result is obtained using the dilated 10 years;

$t_E = \gamma \times 10\ \text{years}$: $\gamma = (1 - u^2/c^2)^{-1/2} = 1.1 \rightarrow t_E = 11\ \text{years}$.

3. Let $O'$ be moving at speed $u$ with respect to $O$. Since the clocks are identical, their period, as measured in their respective rest frames, is the same. Let this period be $\tau$. Each sees the other's clock running a factor $\gamma$

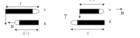
slower than their own clock. Thus far the student's reasoning is sound. The first three events, together with their spacetime coordinates in the respective frames are:

| Event | Spacetime Coordinates in $O$ | Spacetime Coordinates in $O'$ |
|---|---|---|
| 1) Light clock at rest in $O$ emits pulse and light clock at rest in $O'$ emits pulse | $x = 0 \quad t = 0$ | $x' = 0 \quad t' = 0$ |
| 2) Light clock at rest in $O$ completes period. | $x = 0 \quad t = \tau$ | $x' = -\gamma u\tau \quad t' = \gamma\tau$ |
| 3) Light clock at rest in $O'$ completes period. | $x = \gamma u\tau \quad t = \gamma\tau$ | $x' = 0 \quad t' = \tau$ |

Thus, remembering that an 'observer' is really a shorthand for a latticework of rods and clocks, it is the lab clock at location $x = \gamma u\tau$ which records the 'tock' of the rocket clock. This lab clock reads $t = \gamma\tau$ when this event occurs. Since the lab clocks are synchronised in the lab frame, the lattice clock at the origin then also reads $t = \gamma\tau$. Thus Event 4 (the event which, in the lab frame is simultaneous with Event 3, but occurring at the lab origin) occurs some time *after* the return of the lab observer's light clock pulse at that location. The rocket observer assigns $(x' = -\gamma^2 u\tau, t' = \gamma^2\tau)$ to Event 4. Note that in the rocket frame Event 4 occurs precisely $\gamma$ times later than the lab-assigned time for $O'$'s light-clock period ($= \gamma\tau$). At the instant the rocket light pulse returns *all* his clocks read $t' = \tau$ and the fact that Event 4 has yet to occur in his frame accounts consistently (from his viewpoint) for the fact that his lab-based colleague will assign a dilated time. For the rocket observer it is exactly the same, his lattice clock at $x' = -\gamma u\tau$ records the completion of the lab clock's period (Event 2) at rocket time $t' = \gamma\tau$. *All* the rocket observer's clocks read $t' = \gamma\tau$ when this event occurs, including the one at his origin $(x' = 0)$. This event occurring at the rocket origin (Event 5, say), recorded by the rocket observer as $(x' = 0, t' = \gamma\tau)$, is recorded by the lab observer as $(x = \gamma^2 u\tau, t' = \gamma^2\tau)$.

To summarise, each observer correctly concludes that events occurring *at the same place* but at different times in the other observer's frame, have a longer time separation in their own frame, where the events are spatially separated. However, there is no paradox, since there is no single lattice clock with which to effect the comparison to determine who's time is 'really' going slower. The question 'Who is *actually* older than whom?' really doesn't make sense unless the spatial separations between the comparison events is removed. The attempt to remove the spatial separation by having one observer turn around and come back, so that the clock's at the respective origins will once again meet up, is an issue which will be addressed in a subsequent problem.

4. Consider the situation from Spock's viewpoint (left-hand sketch): Kirk's ship will be contracted by factor $\gamma = (1 - u^2/c^2)^{-1/2}$ *so the shot will MISS.*




Now from Kirk's viewpoint (right-hand sketch):
Spock's ship will be contracted by the factor $\gamma$ and Kirk is obviously worried. Indeed, if he were hit this would make a total nonsense of relativity: the ship is either hit, or it is not hit, independent of reference frame.
Let us consider the problem systematically:
There are two events:

Event 1: Spock's nose is level with Kirk's tail.
Event 2: The gun is fired from Spock's tail.

In frame $S$: $(x_1, t_1)_S = (0,0)$ defining the origin, in $S$, at event 1:
$(x_2, t_2)_S = (-L, 0)$ the gun is fired at $-L$
and it fires simultaneously with event 1.

Now in Kirk's frame (travelling at velocity $-u$ relative to Spock)
In frame $K$: $(x_1, t_1)_K = (0,0)$ as origins of $S$, $K$ coincide.
But: $x_2^K = \gamma(-L + u \times 0) = -\gamma L$
And $t_2^K = \gamma(0 + u/c^2(-L)) = -\gamma uL/c^2$
i.e. event 2 is *earlier* than event 1 (according to $K$) and the gun fires at $-\gamma L$ which is *ahead* of Kirk's nose (at $-L$).
So the situation, according to Kirk, in fact looks like this. The shot clearly MISSED.