First Year **Relativity** **Problem Sheet 1** 09Dec04

1. (a) At what speed is a metre rule moving relative to an observer who finds its length only 750mm?
(b) According to Mr Spock's clock, Captain Kirk's clock is running slow by 5%. What is their relative speed?
(c) Plot a graph of the Lorentz factor, $\gamma = (1 - u^2/c^2)^{-1/2}$, as a function of u/c from 0 to 1 in steps of 0.1. Above what speed are relativistic corrections to Galilean kinematics more than 10%?
(d) Use the binomial expansion to find an approximation for γ when $u << c$, up to terms of order $(u/c)^2$

(Answers (a) 0.66c, (b) 0.3c, (c) ~0.4c)

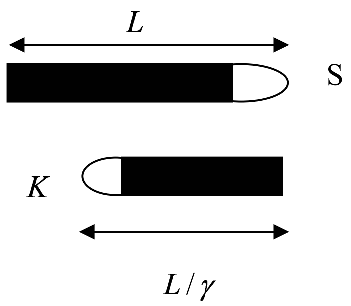
2. Alpha Centauri is 4.4 light years away (as measured in the Earth frame of reference). If astronauts wanted to reach it in ten years as reckoned by their own clocks, at what (constant) speed would they need to travel? How long would the trip take according to Earth time?

3. A student complained after one of my lectures that the theory of relativity must be wrong because using the theory one can argue as follows: A light clock at rest in the rocket frame O' is seen to go slow by an observer at rest in the laboratory frame O. Similarly a light clock at rest in O is observed by O' to go slow. Both clocks cannot be 'going slow', hence contradiction. Define coordinate systems S and S' as usual in such a way that the coordinate axes overlap at $t = t' = 0$. Consider the space and time coordinates of the 4 events:

1) light pulses are emitted by light clocks respectively at rest in O and O' at space, time coordinates $(t, x) = (0,0)$ and $(t', x') = (0,0)$;
2) light clock at rest in O completes one period;
3) light clock at rest in O' completes one period; and
4) the event which, in the lab frame of reference, is simultaneous with 3), but located at the lab origin,

and hence show how the 'paradox' is resolved.

4. *Hit or Miss?* Captain Kirk (K) and Mr Spock (S) pilot identical spaceships of length L (in their own respective rest frames). A prearranged practice session involves K and S approaching each other, almost head-on with high relative speed, u. At the instant when the *nose* of S's ship is level with the *tail* of K's ship then S will fire a warning shot from a gun in his tail in front of K (see figure). S has reassured K that this is harmless due to the Lorentz contraction of K's ship to a length L/γ as illustrated. Imagine K's worried expression when approaching head-on and observing S's ship contracted. Ignore transverse separation of ships and bullet travel times. Does the bullet hit or miss? Analyse (using Lorentz transformations) the problem from both Kirk's and Spock's viewpoint.



Practice session from S's viewpoint