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Relativity – Lecture 7

Energy and momentum – applications II

Lecture 7: *E* & *p*: Applications II

7.1 The photon

- The photon is the quantum of light:
  - Travels at the speed of light
  - Rest mass of photon is zero …
    it has no rest frame
- Energy of photon of frequency f is $E_\gamma = hf$
  where h is Planck's constant
- Photon momentum:
  - Invariant-mass formula applied to photon:

$$\left(m_0 c^2\right)^2 = E_\gamma^2 - \left(cp_\gamma\right)^2 = (hf)^2 - \left(cp_\gamma\right)^2 = 0$$

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- Rearrange to give: $p_\gamma = \frac{hf}{c} = \frac{h}{\lambda}$ (wavelength = λ)
- Red-shift formula using energy-momentum Lorentz transformation:
  - In S' photon frequency f' (parallel to –ve x' axis)
  - In S photon frequency f (parallel to –ve x axis)

$$E_\gamma = \gamma\left(E'_\gamma + \beta\left(-cp'_\gamma\right)\right)$$

$$hf = \gamma\left(hf' - \beta c \frac{hf'}{c}\right)$$

$$f = f'\sqrt{\frac{1-\beta}{1+\beta}}$$

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7.2 Energy/momentum conservation

- Total (relativistic) energy of system of particles conserved
- Total (relativistic) momentum of system of particles conserved
- Consequence:
  - Can turn rest-mass energy into kinetic energy
  - Can turn kinetic energy into rest-mass energy (i.e can turn energy of motion into matter)

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7.3 Natural units

- Speed of light 'natural' unit of speed
  - Set $c = 1$
- Energy gained by electron accelerated through potential of 1 V → 'natural' unit of energy
  - i.e. 1 eV = energy gained by electron accelerated through potential of 1 V
  - $1\,\text{eV} = 1.6\times10^{-19}$ J
  - Natural unit of mass: eV/c^2
  - Natural unit of momentum: eV/c

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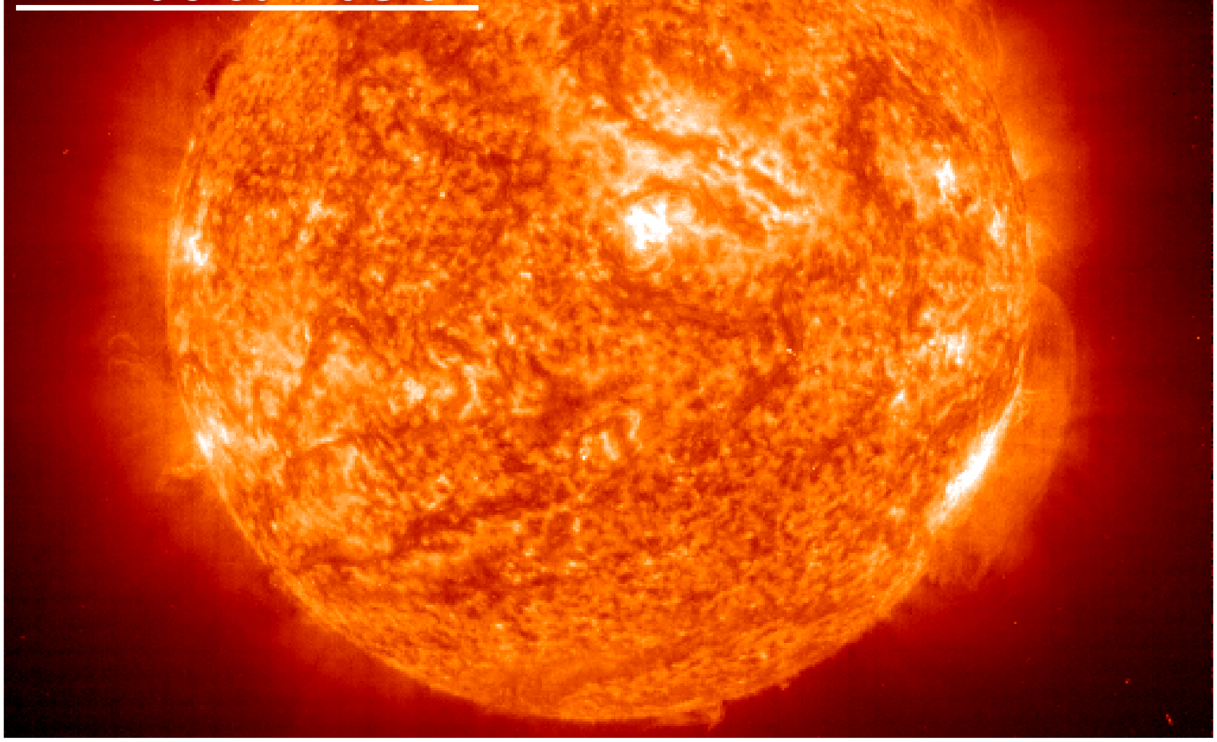
7.4 Nuclear fusion



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pp	$\rightarrow$	$^2H + e^+ + \nu_e$	0.42 MeV	
$^2H + p$	$\rightarrow$	$^3He + \gamma$	5.5 MeV	
$^3He + {}^3He$	$\rightarrow$	$^4He + 2p$	12.8 MeV	85%
$^3He + {}^4He$	$\rightarrow$	$^7Be + \gamma$		15%
$e^- + {}^7Be$	$\rightarrow$	$^7Li + \nu_e$		
$^7Li + p$	$\rightarrow$	$2\,{}^4He$		
$p + {}^7Be$	$\rightarrow$	$^8B + \gamma$		0.02%
^8B	$\rightarrow$	$^8Be^* + e^+ + \nu_e$		
$^8Be^*$	$\rightarrow$	$2\,{}^4He$		

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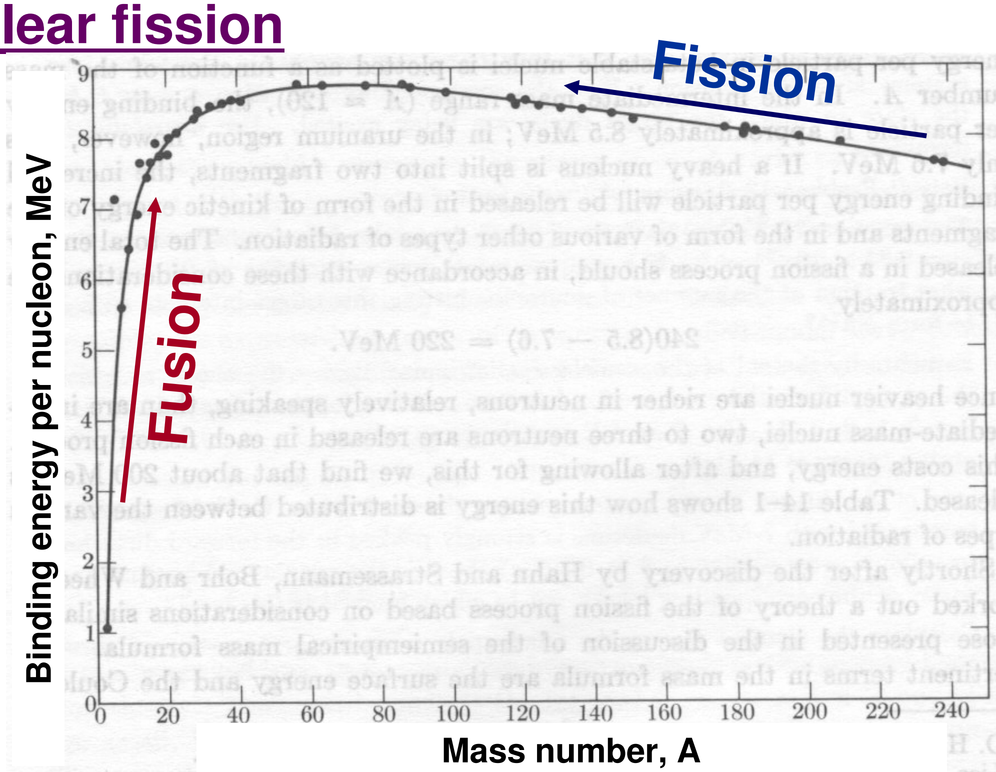
7.5 Nuclear fission



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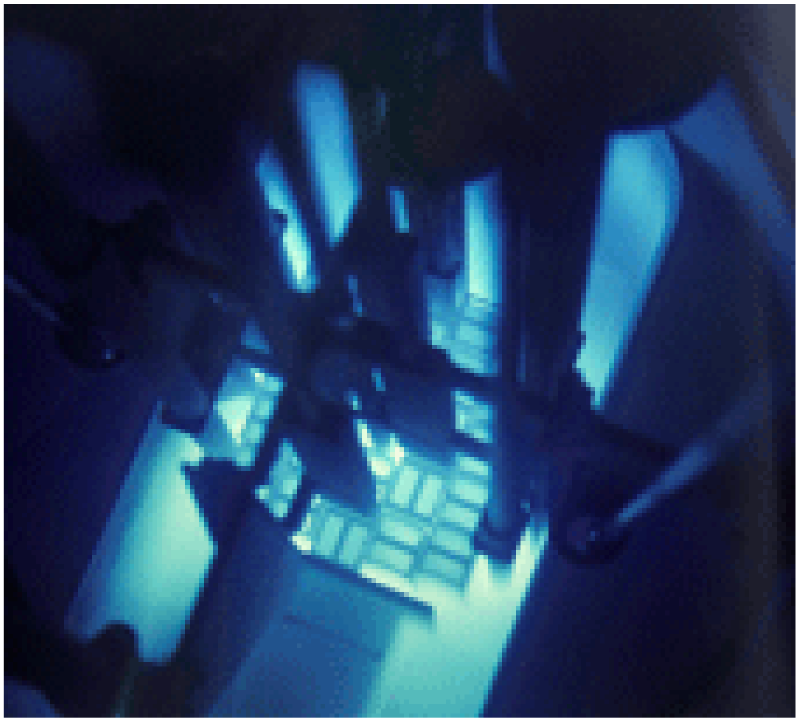
- Silwood Park: Imperial College Reactor Ctr



Small reactor used for research (e.g. effect of radiation on materials)

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7.6 Decay of the K^0 meson

- Rest mass of K^0 = 500 MeV/c²
- Decay: $K^0 \rightarrow \pi^+ + \pi^-$
- Consider:
  - K^0 to be at rest in S'
  - Decay products to be produced parallel to the y' axis
  - Use energy/momentum conservation to show: $E'_\pi = 250\ \text{MeV}$
  - Momentum conservation implies: $p'_{y_{\pi^+}} = p'_{y_{\pi^-}} = p'_{y_\pi}$
  - Calculate pion momentum: $p'_{y_\pi} = \sqrt{E'^2_\pi - m^2_\pi} = 207\ \text{MeV/c}$

Using 'natural' units!

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- Now consider K^0 to be moving so that its energy is 2000 MeV
  - Take K^0 rest frame (S') to be moving relative to S (usual configuration)
  - Calculate γ and β: $E_K = \gamma m_K = 2000 = \gamma \times 500$

$$\Rightarrow \gamma = 4$$

$$\Rightarrow \beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = \frac{\sqrt{15}}{4}$$

Using 'natural' units!

  - Lorentz transformation now gives E_π and $p_{x\pi}$:

$$E_\pi = \gamma\left[E'_\pi + \beta p'_{x_\pi}\right] = 4 \times 250 = 1000\ \text{MeV}$$

$$p_{x_\pi} = \gamma\left[p'_{x_\pi} + \beta E'_\pi\right] = 4 \times \frac{\sqrt{15}}{4} \times 250 = 968\ \text{MeV}/c$$

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- Coordinates transverse to the relative motion do not transform:
  - $p_{y_\pi} = p'_{y_\pi}$

    $p_{z_\pi} = p'_{z_\pi}$

- Check invariant mass evaluated in S:

$$\begin{aligned}[m_K]^2 &= (E_{\text{Tot}})^2 - (p_{x\text{Tot}})^2 - (p_{y\text{Tot}})^2 - (p_{z\text{Tot}})^2 \\ &= (2000)^2 - (250\sqrt{15})^2 - 0 - 0 \\ &= 4 \times 10^6 - 3.75 \times 10^6 = 25 \times 10^4 \\ &= [500]^2\end{aligned}$$