K. Long, 13 December 2004

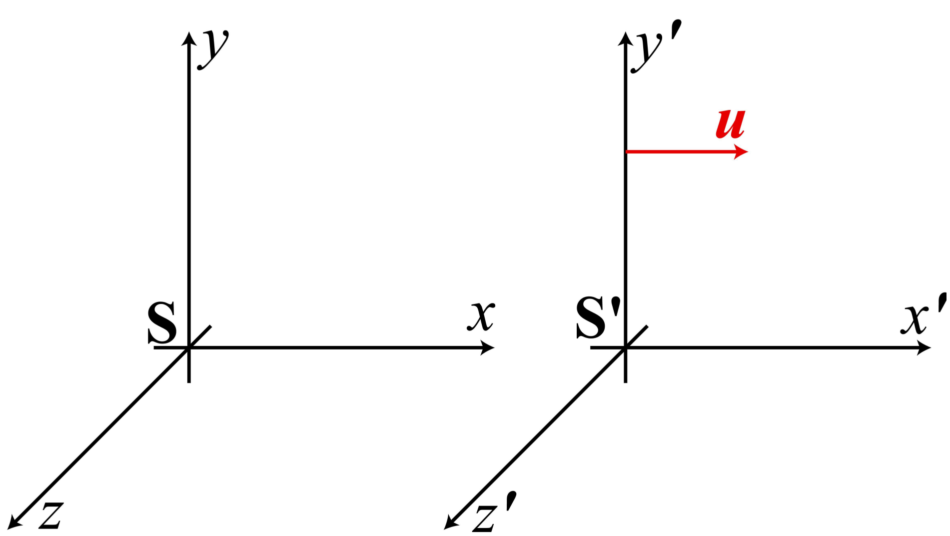
Relativity – Lecture 6

Energy and momentum

Lecture 6: Energy and moment^m

6.1 Space-time diagram

- Usual definition of *S* and *S*′:
  - ‘Standard’ diagram shows space coordinates



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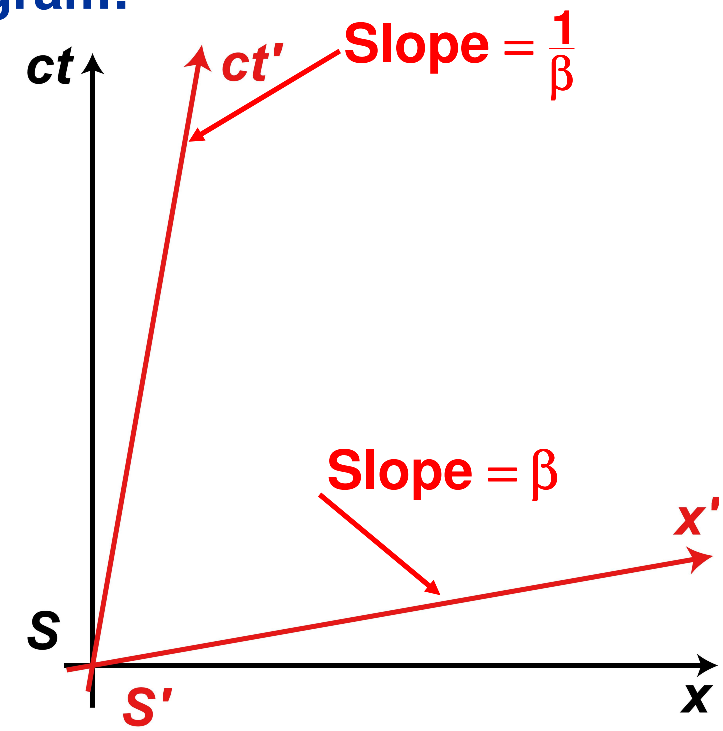
- Space-time diagram:
  - Attempt to show position and time coordinates of events



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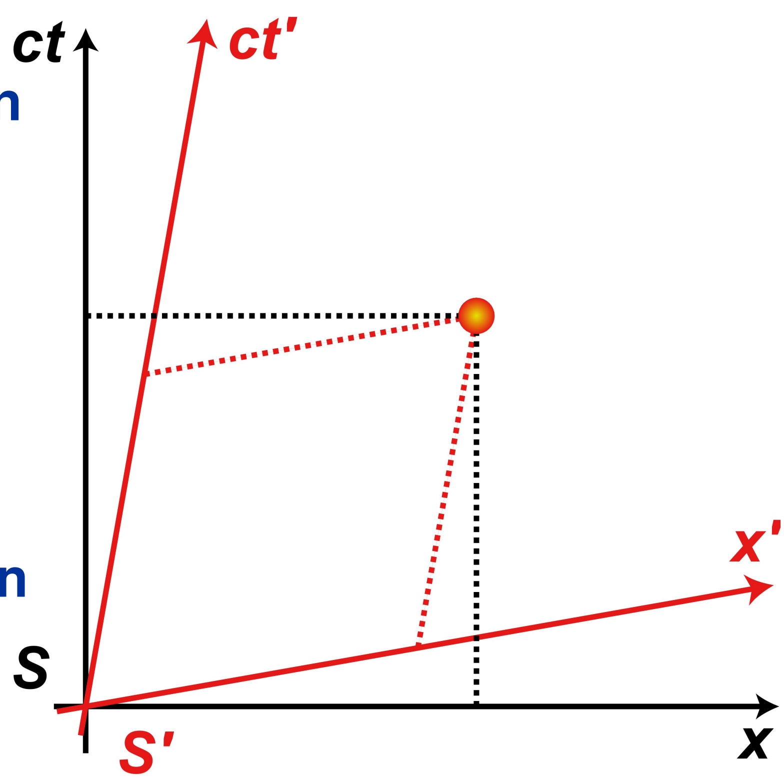
- Pictorial representation of space-time position of event in *S* and *S´*:
- Mathematical connection in Lorentz transformation equations



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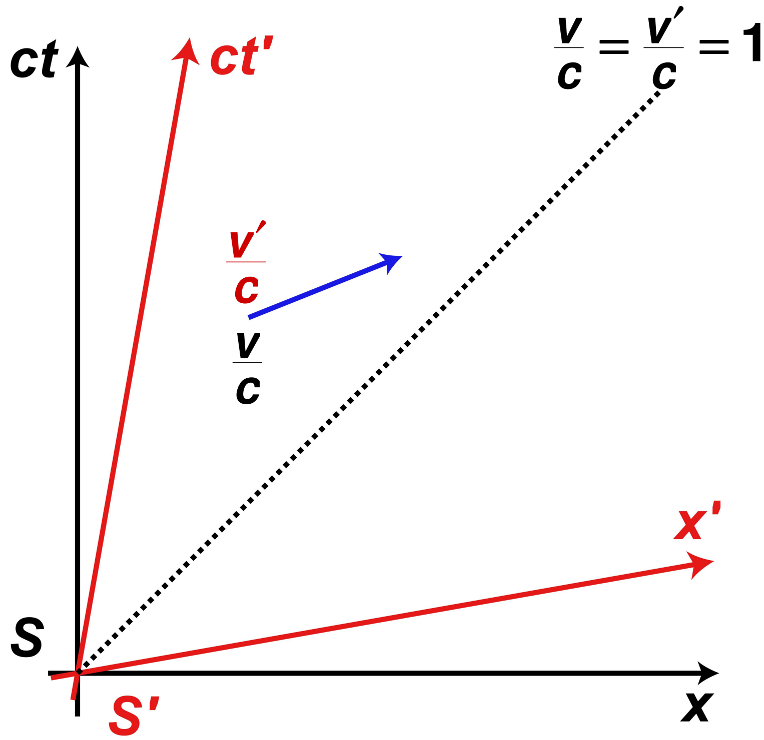
- Pictorial representation of velocity transformation
- Mathematical connection in velocity transformation equations



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6.2 Non-relativistic kinetic energy and momtm

- Non-relativistic formula

Momentum	Kinetic energy
$\underline{p} = m\underline{v}$	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

- Maximum speed *c* implies maximum ***non-relativistic*** momentum and kinetic energy …
  - Routinely falsified at accelerator research laboratories [CERN, DESY, FNAL, SLAC, KEK, Novosibirsk, …]

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6.3 The rest frame

- **Proper time:**
  - Time interval between two events that occur *at same position* – in 'rest frame'
- **Proper length:**
  - Length of object in frame in which object is at rest – 'rest frame'
- **Rest mass:**
  - Mass of object in frame in which object is at rest – 'rest frame'
  - Define: m_0 = rest mass

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6.4 Lesson from correspondence principle

- **Correspondence principle:**
  - Object with small velocity ($v << c$) must satisfy

$$p = m_0 v$$

- **Relativistic definition of momentum must reduce to this when $v << c$**
  - So, seek definition of momentum that satisfies:

$$p \propto m_0 \text{ and } p \propto v$$

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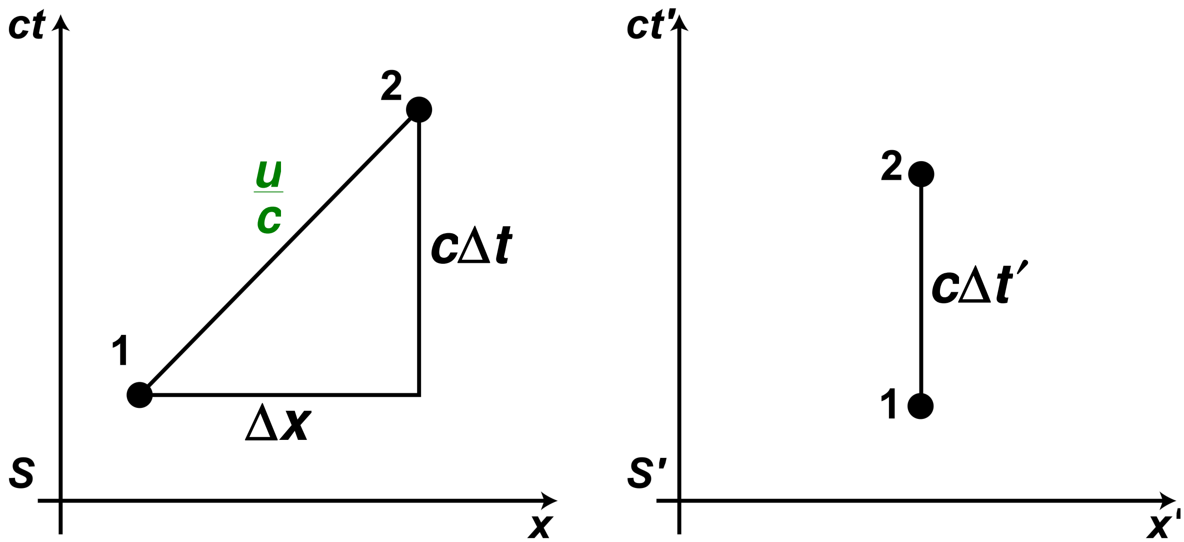
6.5 Lesson from the space time diagram

- Consider object *at rest* in S': rest mass m_0



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- Invariant interval:

$$[c\Delta t']^2 = [c\Delta t]^2 - [\Delta x]^2$$

- Manipulate this equation as follows:
  - Multiply both sides by $\left[m_0 c^2\right]^2$
  - Divide both sides by $[c\Delta t']^2$
  - Apply time-dilation formula $c\Delta t = \gamma c \Delta t'$
  - Rearrange to give:

$$\left[m_0 c^2\right]^2 = \left[\gamma m_0 c^2\right]^2 - \left[\gamma\beta m_0 c^2\right]^2 \qquad (6.1)$$

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6.6 Relativistic definition of momentum

- Analogy with space and time:
  - Space-time:

Space-time:	time	ct	
	position	r	'3 vector'
Mmtm-energy:	energy	E	
	mmtm	p	'3 vector'

- Analogue of invariant interval:

$$\left[m_0c^2\right]^2 = E^2 - (cp)^2 \qquad \mathbf{6.2}$$

- Definition of momentum:
  by comparison of 6.1 with 6.2

$$cp = \gamma\beta m_0 c^2$$

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6.7 Relativistic definition of energy

- Relativistic definition of energy:
  by comparison of 6.1 with 6.2

$$E = \gamma m_0 c^2$$

6.8 Relativistic definition of *E* and *p*: summary

- Energy: $E = \gamma m_0 c^2$
- Momentum: $p = \gamma\beta m_0 c$
- Invariant (rest) mass: $\left[m_0c^2\right]^2 = E^2 - (cp)^2$

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6.9 Lorentz transformation of energy and momentum

Transformation	Inverse transformation
$E' = \gamma(E - \beta c p_x)$	$E = \gamma(E' + \beta c p'_x)$
$cp'_x = \gamma(cp_x - \beta E)$	$cp_x = \gamma(cp'_x + \beta E')$
$cp'_y = cp_y$	$cp_y = cp'_y$
$cp'_z = cp_z$	$cp_z = cp'_z$