K. Long, 2 December 2004

Relativity – Lecture 4

The Lorentz transformation

Lect. 4: The Lorentz transformat[n]

4.1 (More) definitions

- Invariant interval links space coordinates to the time coordinate *mutliplied by the speed of light.*
- Choose to use *ct* for the time coordinate so that time is 'measured' in metres and so treated like the space dimensions
- So: space time coordinates:
  - In S: (ct, x, y, z)
  - In S′: (ct', x', y', z')

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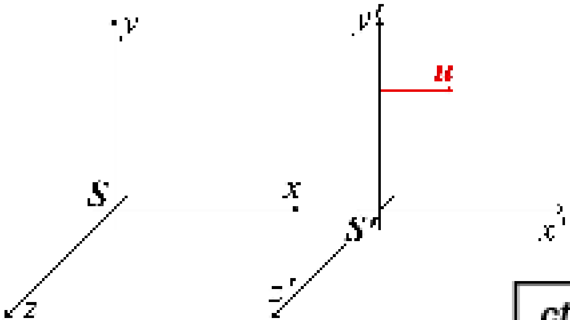
4.2 Form of the solution

- Transformation equations must be ***LINEAR***
  - To ensure solutions are unique
  - To ensure principle of relativity is satisfied



$$
\begin{aligned}
ct' &= g(ct, x, y, z) = b_1 ct + b_2 x + b_3 y + b_4 z \\
x' &= f(ct, x, y, z) = a_1 ct + a_2 x + a_3 y + a_4 z \\
y' &= y \\
z' &= z
\end{aligned}
$$

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4.3 Apply principle of relativity

- Consider body moving parallel to y axis in S
  - Implies $a_3 = b_3 = 0$
- Consider body moving parallel to z axis in S
  - Implies $a_4 = b_4 = 0$

$$
\begin{aligned}
ct' &= g(ct, x, y, z) = b_1 ct + b_2 x \\
x' &= f(ct, x, y, z) = a_1 ct + a_2 x \\
y' &= y \\
z' &= z
\end{aligned}
$$

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4.4 View *S* from *S′*; apply time dilation

- Locus of point (*x*,*y*,*z*) = (0,0,0) seen from *S′*

$$x' = -\beta(ct')$$

- Consider two events *both* at (*x*,*y*,*z*) = (0,0,0) but which occur at different times

$$ct' = \gamma(ct)$$

- Substitute/rearrange to show: $a_1 = -\beta\gamma$ $b_1 = \gamma$

$$\begin{aligned}
ct' &= g(ct, x, y, z) = \gamma ct + b_2 x \\
x' &= f(ct, x, y, z) = -\beta\gamma ct + a_2 x \\
y' &= y \\
z' &= z
\end{aligned}$$

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4.5 View *S′* from *S*

- Locus of p^{nt} (*x′*,*y′*,*z′*) = (0,0,0) seen from *S*

$$x = \beta(ct)$$

- Substitute/rearrange to show: $a_2 = \gamma$

$$\begin{aligned}
ct' &= g(ct, x, y, z) = \gamma ct + b_2 x \\
x' &= f(ct, x, y, z) = -\beta\gamma ct + \gamma x \\
y' &= y \\
z' &= z
\end{aligned}$$

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4.6 The speed of light is invariant

- Consider light pulse set off along +ve x (x') axis at $t = t' = 0$
- Show light pulse propagation satisfies:

$$\frac{x}{ct} = 1 = \frac{x'}{ct'}$$

- And hence that: $b_2 = -\beta\gamma$

$$\begin{aligned} ct' &= g(ct, x, y, z) = \gamma ct - \beta\gamma x \\ x' &= f(ct, x, y, z) = -\beta\gamma ct + \gamma x \\ y' &= y \\ z' &= z \end{aligned}$$

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4.7 The Lorentz transformation

Transformation	Inverse transformation
$ct' = \gamma(ct - \beta x)$	$ct = \gamma(ct' + \beta x')$
$x' = \gamma(x - \beta ct)$	$x = \gamma(x' + \beta ct')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$

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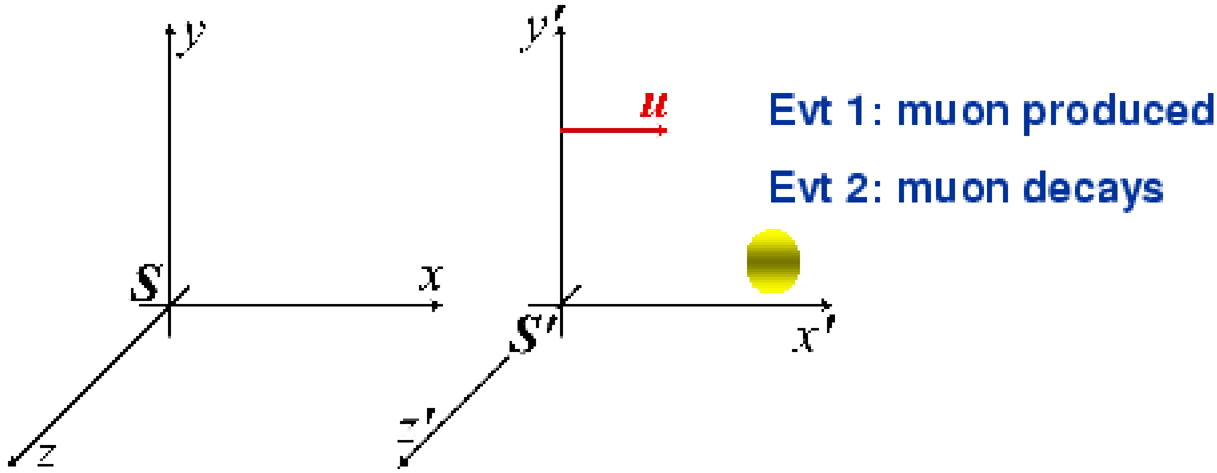
4.8 Examples

- **Time dilation:**
  - Consider two events which occur at the same point in *S′*: eg. production and decay of muon



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4.8 Examples (continued)

- **Length contraction:**
  - Consider two events which occur at the same time in *S*: eg. position at $t = 0$ of the two ends of a rod that is stationary in *S′*: