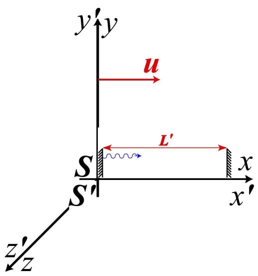
K. Long, 29 November 2004

Relativity – Lecture 3

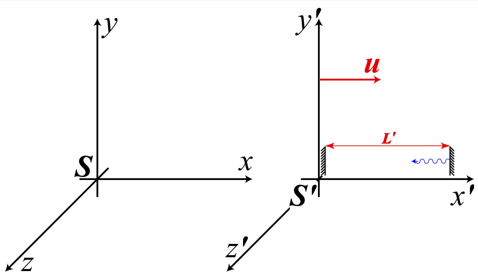
Space and time

Lecture 3: Space and time

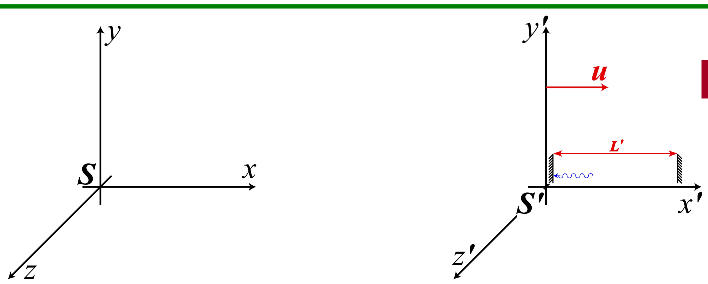
3.1 Length contraction



Event 1: light pulse sets off



Event 2: light pulse is reflected



Event 3: light pulse returns

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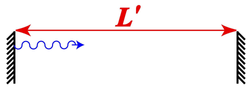
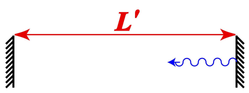
- Analyse from point of view of *O* in *S*´:
  - Event 1: Light pulse sets off:



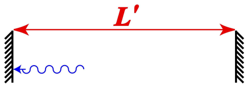
$$t'_1 = 0 \qquad x'_1 = 0$$

  - Event 2: Light pulse reflected:



$$t'_2 = \frac{L'}{c} \qquad x'_2 = L'$$

  - Event 3: Light pulse returns:



$$t'_3 = \frac{2L'}{c} \qquad x'_2 = 0$$

  - 'Round-trip' time in *S*´:

$$T' = \frac{2L'}{c}$$

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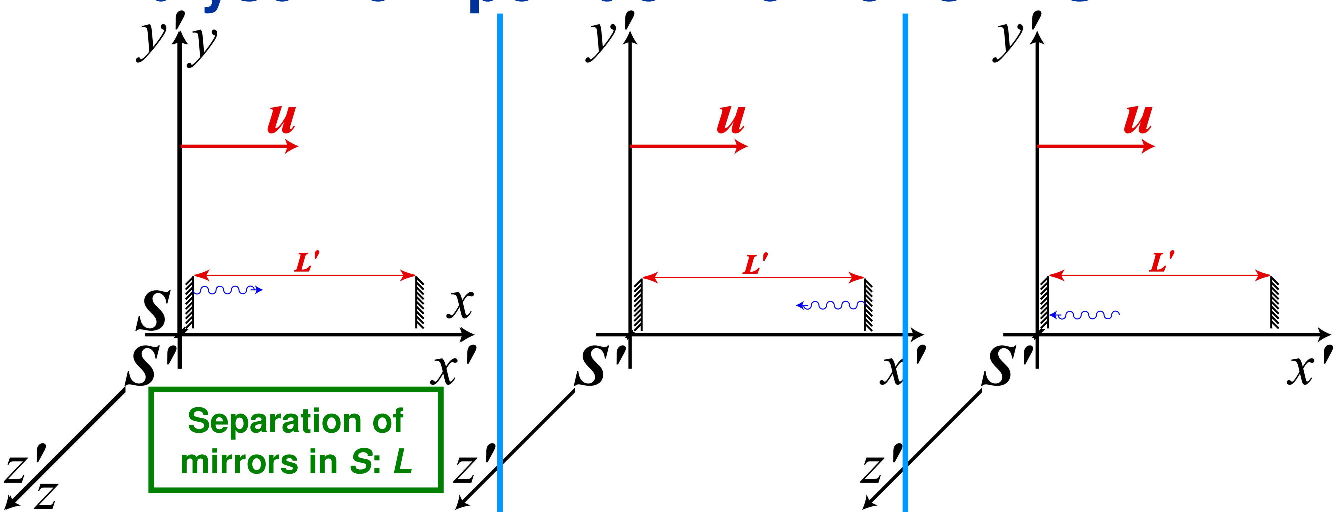
- Analyse from point of view of *O* in *S*:



Event 1:	Event 2:	Event 3:
$t_1 = 0$	$t_2 = \frac{L}{c} + \frac{ut_2}{c}$ $\Rightarrow t_2 = \frac{L}{c-u}$	$t_3 = t_2 + \left[\frac{L}{c} - \frac{u(t_3 - t_2)}{c}\right]$ $\Rightarrow (t_3 - t_2) = \frac{L}{c+u}$

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- **Round-trip time in *S*:** $T = \frac{L}{c-u} + \frac{L}{c+u} = L\left[\frac{c+u+c-u}{c^2-u^2}\right] = \frac{2L}{c}\frac{1}{1-\frac{u^2}{c^2}}$
  - **i.e:** $$T = \frac{2L\gamma^2}{c}$$
- **In *S´* Event 3 occurs at same position as event 1 … so time dilation formula applies:**

$$T = \gamma T' = \gamma\left(\frac{2L'}{c}\right)$$

- **Substituting:** $$\frac{2L\gamma^2}{c} = \gamma\left(\frac{2L'}{c}\right) \Rightarrow L = \frac{L'}{\gamma}$$

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- **Length contraction:**

$$L = \frac{L'}{\gamma}$$

  - **Length *L* defined at instant *t* = 0 (i.e. when coordinate axes coincide).**
  - **Length contraction formula holds when distance between two events is measured at the same instant.**

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3.2 Relativity of space and time

… summary so far

Time coord	$\Delta t = \gamma \Delta t'$	Time dilation
Coord // to relative mot^{n}	$\Delta x = \frac{\Delta x'}{\gamma}$	Length contraction
Coords ⊥ to relative mot^{n}	$\Delta y = \Delta y'$ $\Delta z = \Delta z'$	

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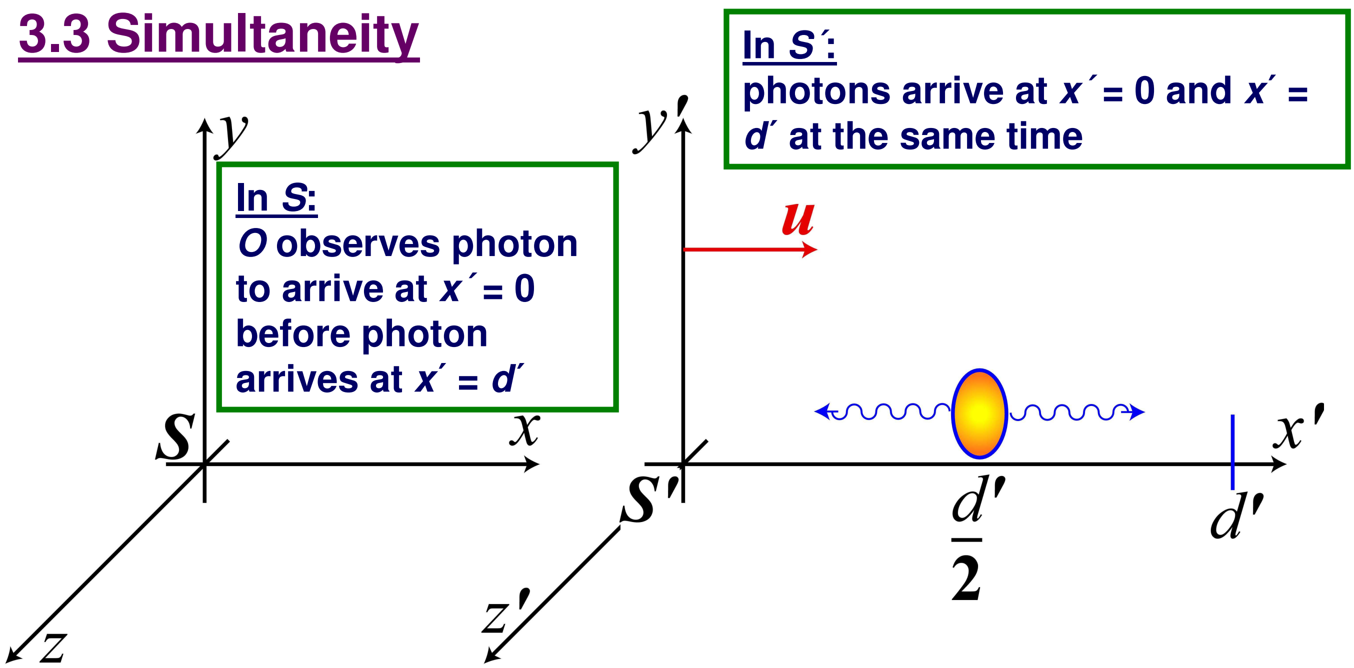
3.3 Simultaneity



- Two events are only simultaneous in *ALL* inertial frames if they take place at the same point in space

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3.4 Proper length and proper time

- **Definition 1:**
  - ***Proper time*:**
    - time difference between two events in inertial frame in which the two events occur at the same position
- **Definition 2:**
  - ***Proper length*:**
    - *length of object in inertial frame in which it is at rest;*
    - *distance between two events in inertial frame in which time interval between two events is zero*

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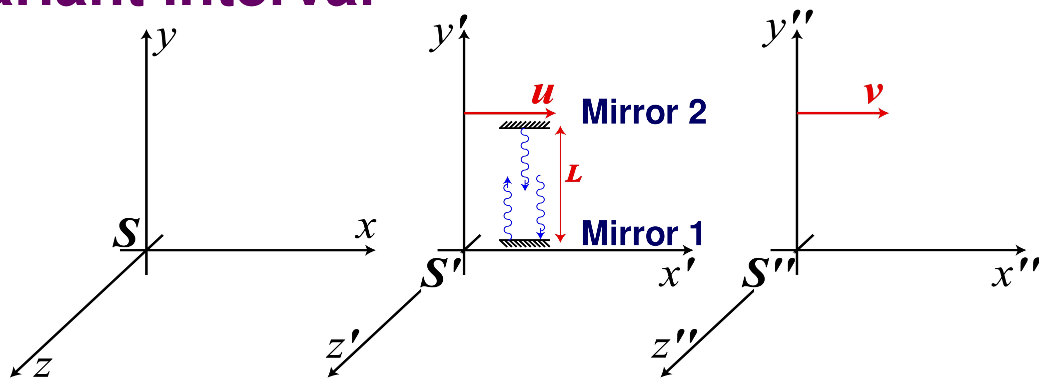
3.5 Invariant interval



- **In each inertial frame *S*, *S*´ and *S*´´, analyse the three events:**
  - **Event 1: Light leaves mirror 1**
  - **Event 2: Light reflected at mirror 2**
  - **Event 3: Light returns to mirror 1**

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- Event 1: Light leaves mirror 1

$$\text{In } S: \quad x_1 = 0 \quad t_1 = 0$$
$$\text{In } S': \quad x_1' = 0 \quad t_1' = 0$$
$$\text{In } S'': \quad x_1'' = 0 \quad t_1'' = 0$$

- Event 2: Light reflected at mirror 2

$$\text{In } S: \quad x_2 \qquad t_2$$
$$\text{In } S': \quad x_2' = 0 \quad t_2' = \frac{L}{c} = \frac{\tau}{2}$$
$$\text{In } S'': \quad x_2'' \qquad t_2''$$

- Event 3: Light returns to mirror 1

$$\text{In } S: \quad x_3 \qquad t_3$$
$$\text{In } S': \quad x_3' = 0 \quad t_3' = \frac{2L}{c} = \tau$$
$$\text{In } S'': \quad x_3'' \qquad t_3''$$

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- Total distance travelled:

$$\text{In } S: \quad 2\sqrt{L^2 + \frac{(x_3 - x_1)^2}{4}} = 2\sqrt{L^2 - \frac{\Delta x^2}{4}}$$
$$\text{In } S': \quad 2\sqrt{L^2 + \frac{(x_3' - x_1')^2}{4}} = 2\sqrt{L^2 - \frac{\Delta x'^2}{4}} \quad = 2L$$
$$\text{In } S'': \quad 2\sqrt{L^2 + \frac{(x_3'' - x_1'')^2}{4}} = 2\sqrt{L^2 - \frac{\Delta x''^2}{4}}$$

- Time taken to travel total distance:

$$\text{In } S: \quad t_3 - t_1 = \Delta t$$
$$\text{In } S': \quad t_3' - t_1' = \Delta t' = \tau$$
$$\text{In } S'': \quad t_3'' - t_1'' = \Delta t''$$

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- Speed of light (constant)

In S: In S': In S'':

$$c = \frac{2\sqrt{L^2 + \frac{\Delta x^2}{4}}}{\Delta t} = \frac{2\sqrt{L^2 + \frac{\Delta x'^2}{4}}}{\Delta t'} = \frac{2\sqrt{L^2 + \frac{\Delta x''^2}{4}}}{\Delta t''}$$

- Rearrange equations, solve for $4L^2$

In S: In S': In S'':

$$4L^2 = c^2\Delta t^2 - \Delta x^2 = c^2\Delta t'^2 - \Delta x'^2 = c^2\Delta t''^2 - \Delta x''^2$$

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- Since coordinates transverse to the relative motion do not transform, can generalise to give *invariant interval*:

$$\begin{aligned} c^2\tau^2 &= c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \\ &= c^2\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2 \\ &= c^2\Delta t''^2 - \Delta x''^2 - \Delta y''^2 - \Delta z''^2 \end{aligned}$$