K. Long, 22 November 2004

Relativity – Lecture 1

Introduction

The need for relativity

Introduction: Aims & objectives

Aims and objectives

Aim:

To give students an understanding of the theory of special relativity and the ability to use the theory to solve problems related to the motion of bodies.

Objectives: already distributed by Ug office:

- be able to state the fundamental postulates of special relativity
- to be able to apply relativistic transformation of position and time
- …
- be able to calculate relativistic energy and momentum in one-dimensional two-body collisions given initial data
- be able to calculate energy and momentum of particles in one frame given their values in another frame

Introduction: reading list

- Young and Freedman, *University Physics*
  Course text. See chapter 37, useful source of problems.
- McCall, *Classical Mechanics*
  Good treatment of mechanics and relativity, clear explanations. See chapters 5 and 6.
- Alonso and Finn, *Physics*
  Excellent all round book. Good treatment of relativity. See chapters 19 and 20.
- Taylor and Wheeler, *Spacetime physics*
  A book by the real experts, and a very good read. Worth looking at for the insight it offers.

Introduction: admin!

- Office hours:
  Monday 13h00-14h00 and Friday 13h00-14h00
  Room 507, Blackett
- Course associates:
  J. Hassard (Room 504, Blackett)
  M. McCall (Room 611, Blackett)

Lecture 1: The need for relativity

1.1 Introduction

- **Relativity:** study of equations that relate observations in one frame to those in a second frame moving relative to the first with a *constant* velocity $\underline{u}$.

1.2 Definition of an inertial frame of reference

- Inertial frame is one in which Newton's first law:
  - A free particle always moves with constant velocity; i.e. without acceleration

  holds.

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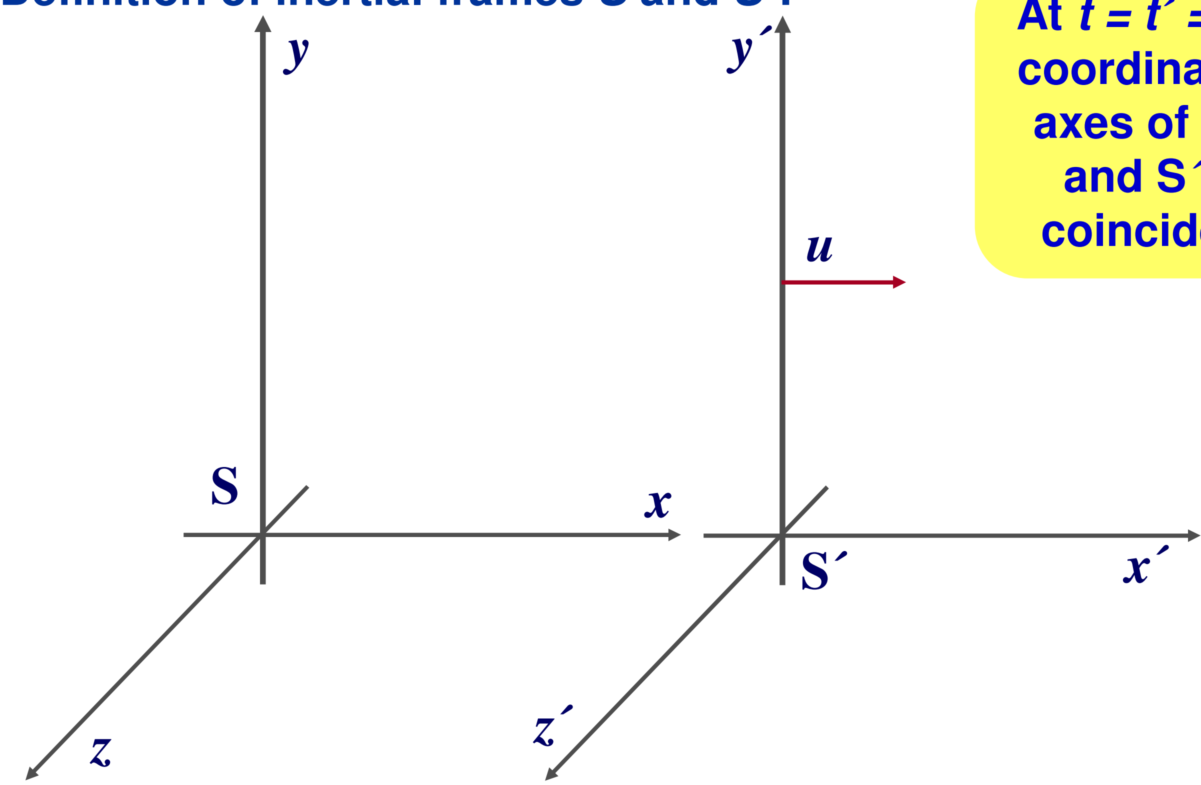
1.3 Galilean relativity [revision!]

- Definition of inertial frames *S* and *S´*:



At $t = t' = 0$ coordinate axes of S and S´ coincide

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- **Galilean transformation of position:**

Transformation	Inverse transf^n
$t' = t$	$t = t'$
$x' = x - ut$	$x = x' + ut'$
$y' = y$	$y = y'$
$z' = z$	$z = z'$

- **Galilean transformation of velocity**

Transformation	Inverse transf^n
$\underline{v}' = \underline{v} - \underline{u}$	$\underline{v} = \underline{v}' + \underline{u}$

- **Galilean transformation holds if:**

$$\underline{v} << c,\ \underline{u} << c \text{ and } \underline{v}' << c$$

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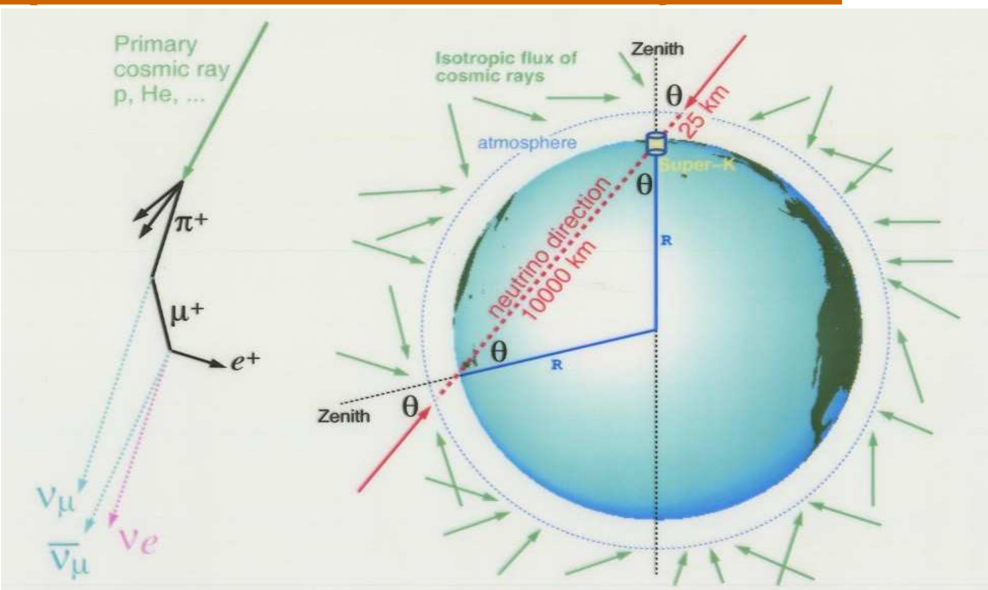
1.4 The problem of the cosmic ray muon



- **Galilean relativity predicts ~100% of muons produced in the upper atmosphere decay before they reach the earth's surface**
  - **In contradiction with observations**

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1.5 The Lorentz transformation

Transformation	Inverse transformation
$ct' = \gamma(ct - \beta x)$	$ct = \gamma(ct' + \beta x')$
$x' = \gamma(x - \beta ct)$	$x = \gamma(x' + \beta ct')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$

$$\beta = \frac{u}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

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1.6 Solution to the cosmic-ray-muon problem

- Assume muon is *stationary* in *S´*
- Time interval between production and decay in *S´*: ΔT´
- Difference in *x´* coordinate between production and decay: Δx´=0
- Hence, using inverse Lorentz transformation obtain time dilation formula:

$$\Delta T = \gamma \Delta T'$$

- Assume earth defines inertial frame *S*, then lifetime of muon in earth frame given by:

$$\text{Lifetime on earth} = \gamma T = 56.6 \times 2.2 = 125\ \mu\text{s}$$

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- Fraction of muons arriving at earth:

$$\text{Fraction arriving at surface} = \exp\left(-\frac{50}{125}\right) \approx 70\%$$

- In agreement with observations