December 11, 2007

Relativity Problem Sheet 3

3.1 Four-Vector Invariants

Prove that

1. $c^2t^2 - x^2 - y^2 - z^2$ and
2. the scalar product of two four vectors

$$a \cdot b \equiv c^2 t_a t_b - x_a x_b - y_a y_b - z_a z_b$$

are invariant under Lorentz transformations.

3.2 Using the Doppler Effect to Check Relativity

Consider an excited atom moving at speed βc in $\mathcal{O}$ and emitting a photon along the $-x$ direction. In the atom's rest frame $\mathcal{O}'$ the photon has a frequency $f' = f_0$ and its energy is $E'_\gamma = p'_\gamma c = hf_0$. Use the the inverse Lorentz Transform for the photon energy and show that its frequency in $\mathcal{O}$ is

$$f = f_0 \sqrt{\frac{1-\beta}{1+\beta}}$$

as expected from the Doppler effect.

3.3 Decay of the B^0

A B^0 decays at rest into a π^+ and a π^-. If $m_{\mathrm{B}^0} = 5279\,\mathrm{MeV}/c^2$, and $m_{\pi^+} = m_{\pi^-} = 139\,\mathrm{MeV}/c^2$ calculate the energy and the momentum of each π.

3.4 Linear Algebra

For the ones who like linear algebra: Write the Lorentz transformation as a 4×4 matrix acting on four-vectors.