December 4, 2007

Relativity Problem Sheet 2

2.1 Speed Limit

If any object moves at $v \leq c$ in any frame, prove that its speed is $v' \leq c$ in any other frame.

2.2 Green-shift of Red Light

Laura is arrested for going through a red light. In court she pleads that since she was driving towards the traffic light the blue-shift caused the red light to appear green. The judge accepts the argument, calculates her speed, and changes the charge to speeding. What is his result?

2.3 Momentum Conservation

Prove that the classical momentum conservation

$$m_1\boldsymbol{v_1} + m_2\boldsymbol{v_2} = m_1\boldsymbol{v_3} + m_2\boldsymbol{v_4}.$$

is not covariant under LT.

Use the special case of an elastic collision of two massive bodies in the centre-of-mass frame where $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ are along the x axis and $\boldsymbol{v_3}$ and $\boldsymbol{v_4}$ after the collision along the y axis. Consider then the same collision in the frame where $\boldsymbol{v_1'} = 0$.

2.4 Practical Units

Express the following quantities in practical units, i.e. eV, MeV/c^2 etc...
($c = 3 \times 10^8\,\mathrm{m\,s^{-1}}, e = 1.6 \times 10^{-19}\,\mathrm{C}$).

(a) Electron mass: $m_e = 9.1 \times 10^{-31}$ kg.

(b) Proton mass: $m_p = 1.67 \times 10^{-27}$ kg.

(c) Total Energy of an electron with momentum $p = 1\,\mathrm{MeV}/c$.

(d) Kinetic Energy of a proton with momentum $p = 1\,\mathrm{MeV}/c$.

(e) Repeat (d) using the Newtonian formula $K = p^2/2m$.

(f) Kinetic Energy of a proton with speed $v = 0.5\,c$.

2.5 The Sun

The Sun produces energy by fusion of hydrogen into helium. One series of processes is:

$$p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu + 0.41\,\mathrm{MeV} \quad (1)$$
$${}^2\mathrm{H} + p \rightarrow {}^3\mathrm{He} + \gamma + 5.51\,\mathrm{MeV} \quad (2)$$
$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p + \gamma + 12.98\,\mathrm{MeV} \quad (3)$$

Calculate the total energy released in the formation of one α-particle (^{4}He nucleus). Note that the e^+(positrons) will annihilate with electrons in the Sun giving extra energy.
Hint: How many times does each of the above reactions need to take place in order to produce a single helium nucleus ?