Relativity — Lecture 8

- **Summary of Lecture 7**
- **Particle kinematics**
- **Four-Vectors introduction**

06/12/2007

Imperial College London
100 years of living science

Patrick Koppenburg

Lecture 7

Revision

Classical Momentum

The classical momentum conservation in an elastic collision says:

$$m_1 \boldsymbol{v}_{\boldsymbol{1}}^{\text{in}} + m_2 \boldsymbol{v}_{\boldsymbol{2}}^{\text{in}} = m_1 \boldsymbol{v}_{\boldsymbol{1}}^{\text{out}} + m_2 \boldsymbol{v}_{\boldsymbol{2}}^{\text{out}}.$$

But this is not covariant under LT. (Problem 2.3).

We need to redefine the momentum to preserve the law. We need

1. A definition of the momentum $\boldsymbol{p}$ such that it is conserved.
2. The low-speed limit must be $\boldsymbol{p} = m\boldsymbol{v}$.
3. The conservation laws must be covariant under LT.

Relativistic Momentum

Definition — Momentum:

$$\boldsymbol{p} = \frac{m\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p_1' + p_2' = \underbrace{\gamma\left(p_1 + p_2\right)}_{\text{constant}} - \underbrace{\gamma\frac{u}{c^2}}_{\text{constant}}\left(\frac{m_1c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2c^2}{\sqrt{1-\frac{v_2^2}{c^2}}}\right).$$

Relativistic Momentum

Definition — Momentum:

$$\boldsymbol{p} = \frac{m\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

The momentum is conserved provided this is conserved:

$$\frac{m_1 c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{m_2 c^2}{\sqrt{1-\frac{v_2^2}{c^2}}}$$

Relativistic Energy

Definition — Energy:

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

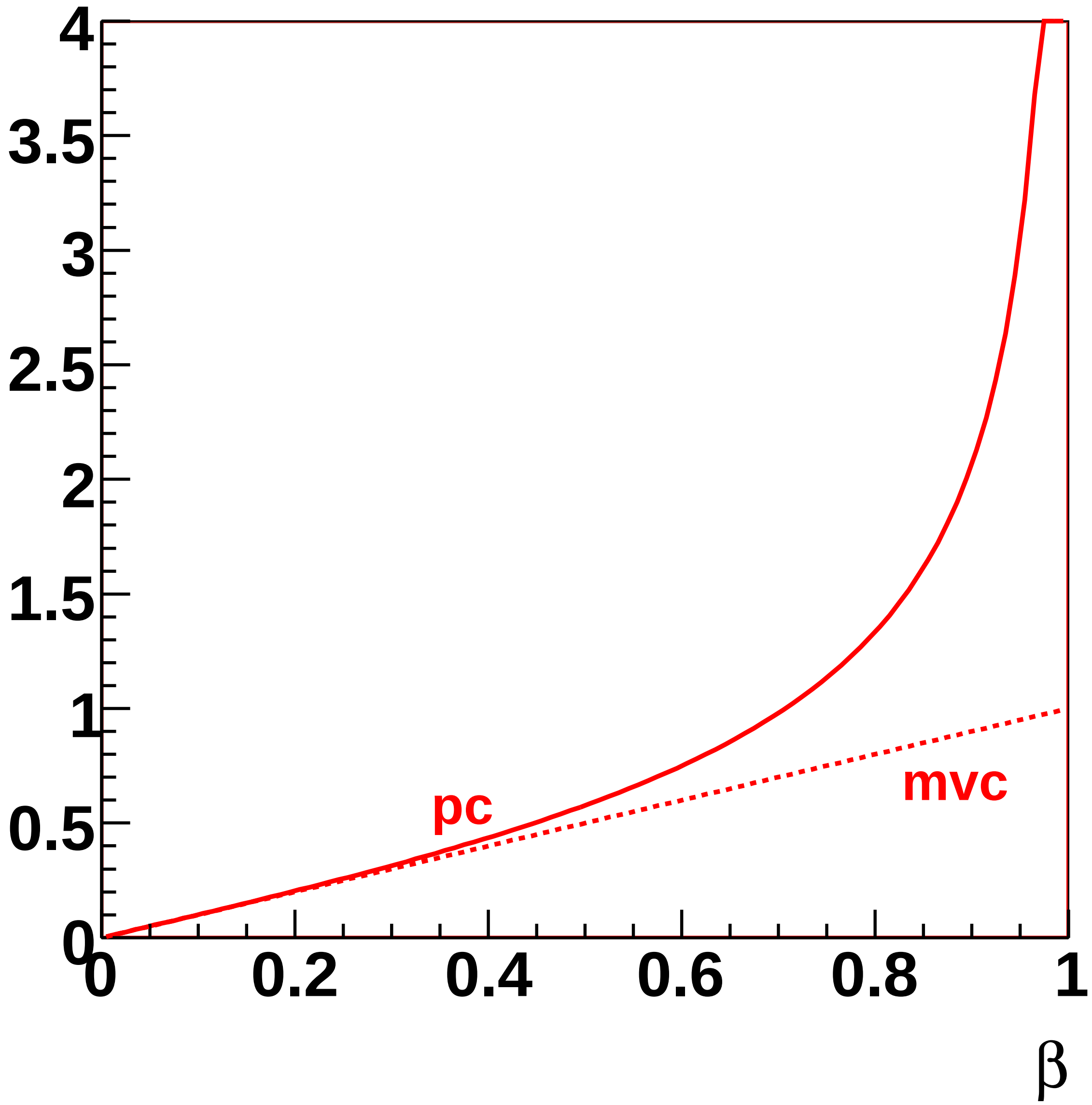
Low speed approximation

Momentum:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

at $v \ll c$:

$$p \simeq mv$$



Low speed approximation

Energy:

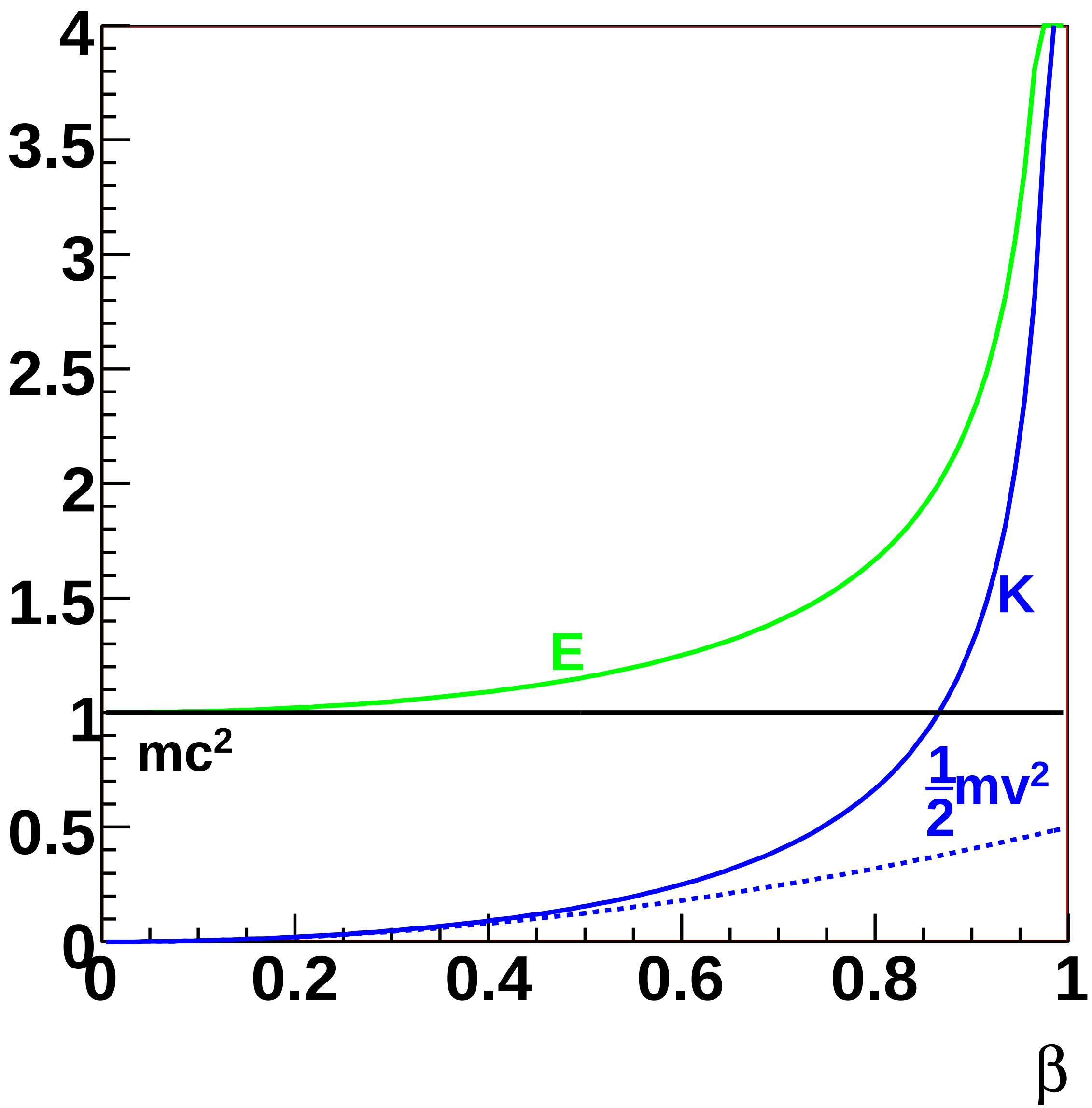
$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Rest energy:

$$E_0 = mc^2$$

Kinetic Energy:

$$K = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$$
$$\simeq \frac{1}{2}mv^2 \quad (v \ll c)$$



Energy-Momentum Relations

For an object with momentum p:

$$E^2 = p^2c^2 + m^2c^4$$

For an object at rest:

$$E_0 = mc^2$$

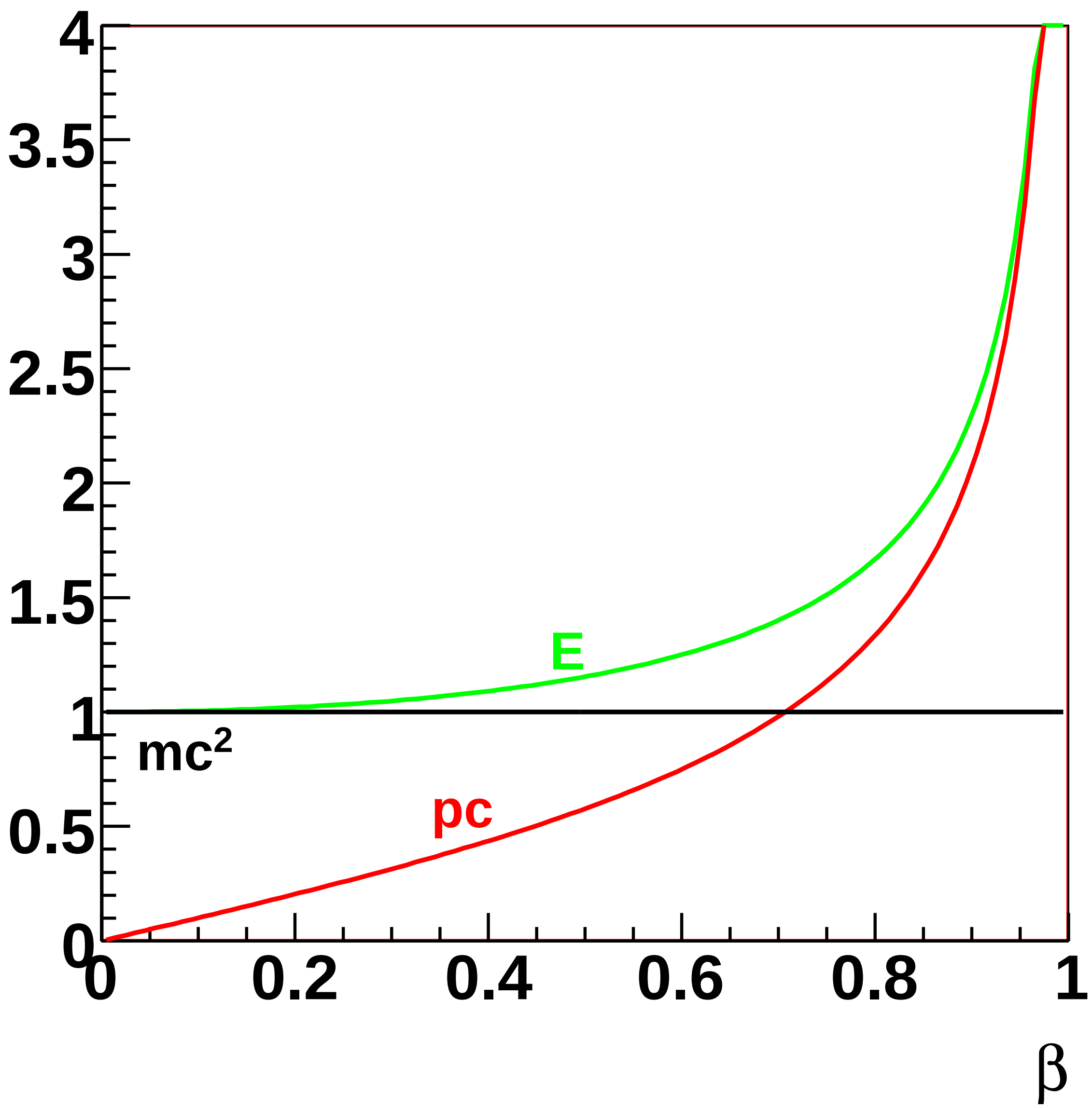
High-speed limit

Energy:

$$E^2 = p^2c^2 + m^2c^4$$

at $v \simeq c$:

$$\Rightarrow \qquad E \simeq pc$$



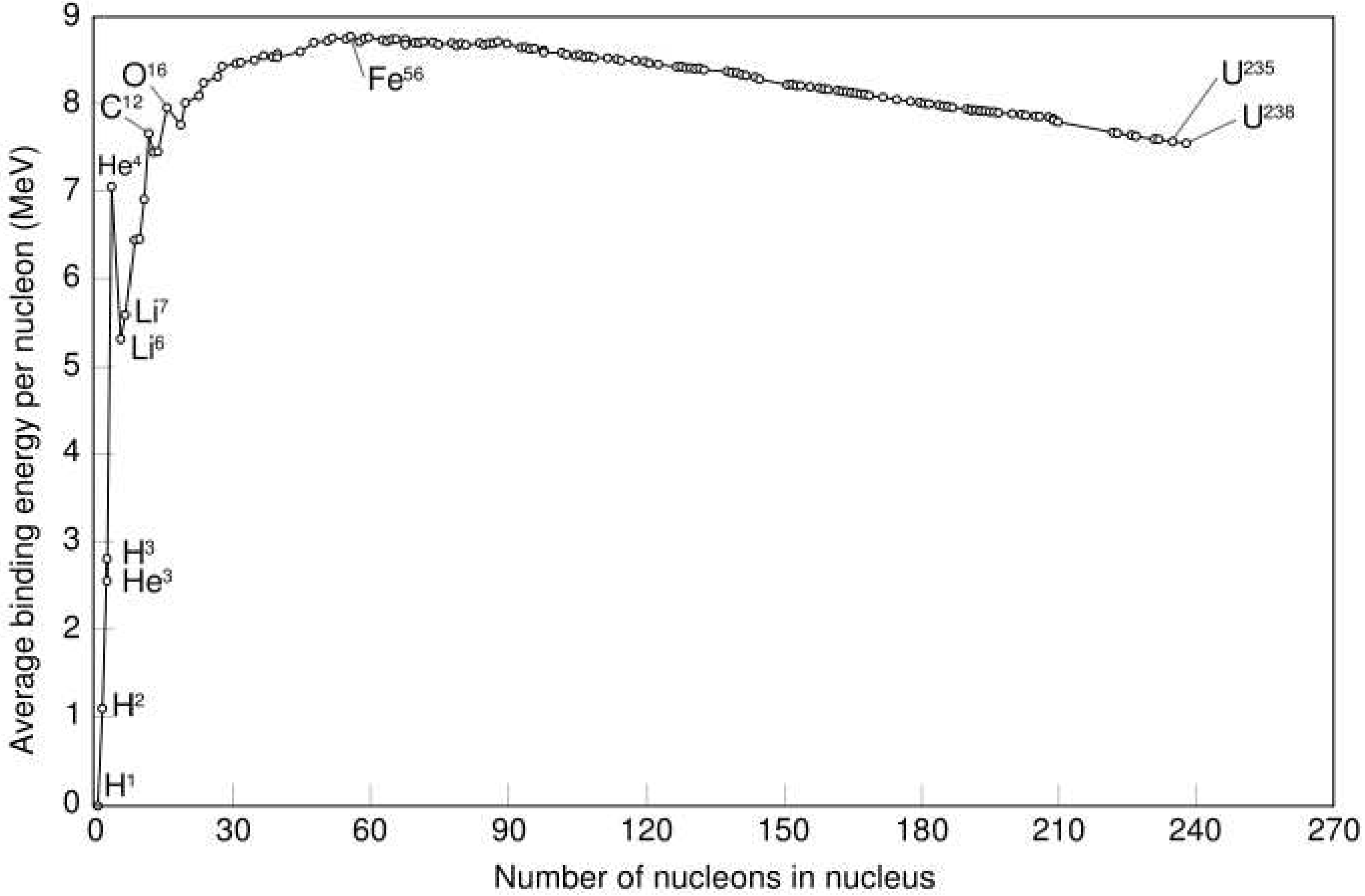
Lecture 8

Electron Gun

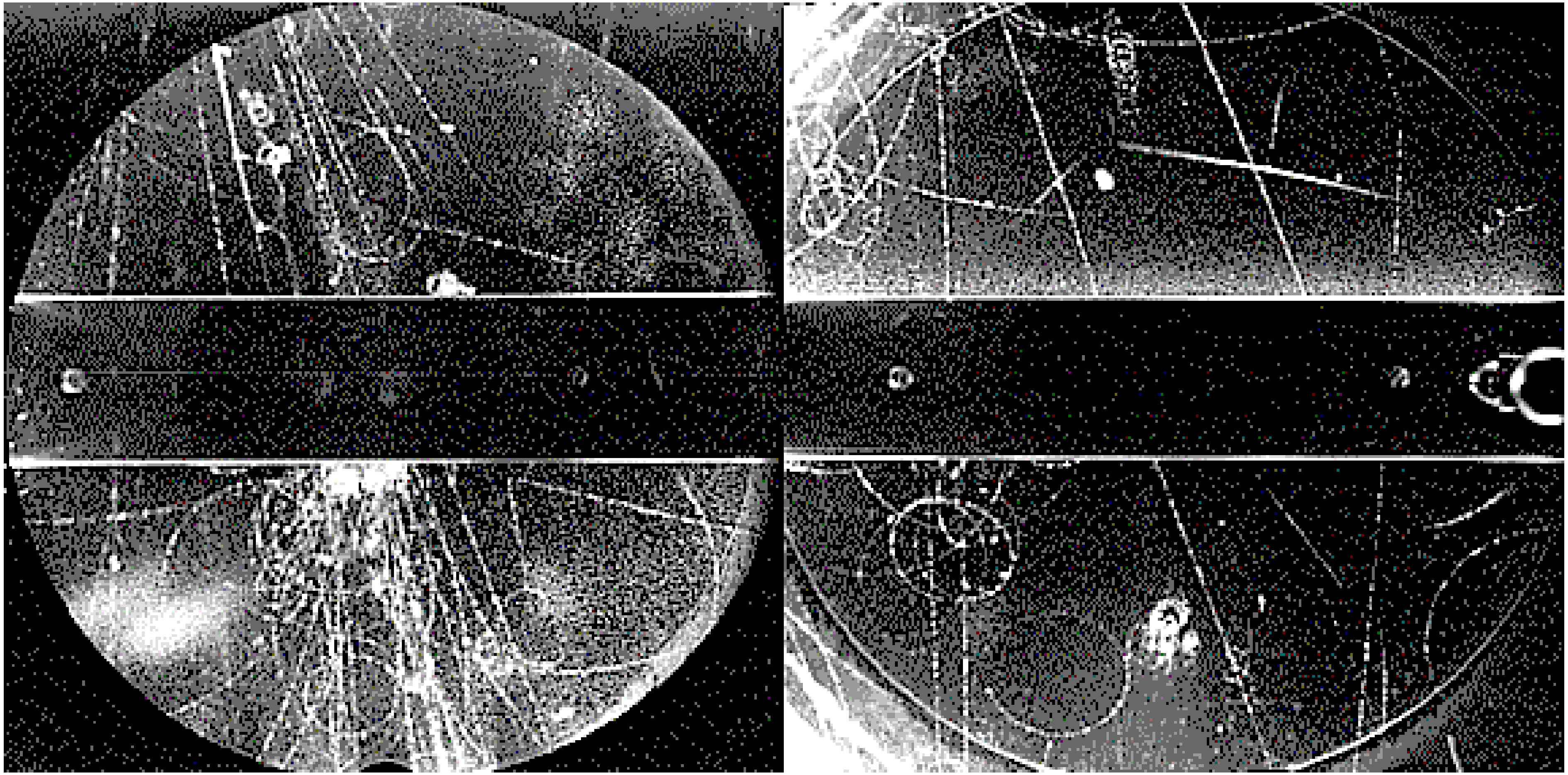
Electrons accelerated to $1\,\mathrm{MV}$

$$\begin{aligned} m_\mathrm{e} &\sim 10^{-30}\,\mathrm{kg} \\ e &= 1.6 \cdot 10^{-14}\,\mathrm{C} \end{aligned}$$

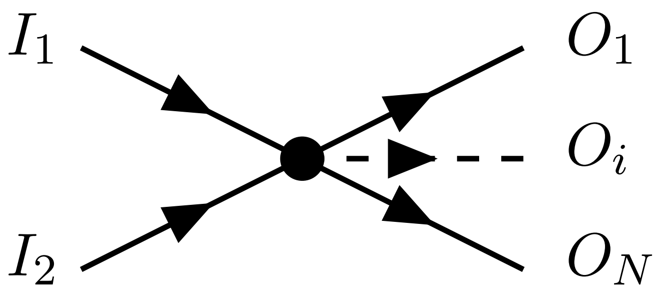
Binding Energy



Kaon Decay

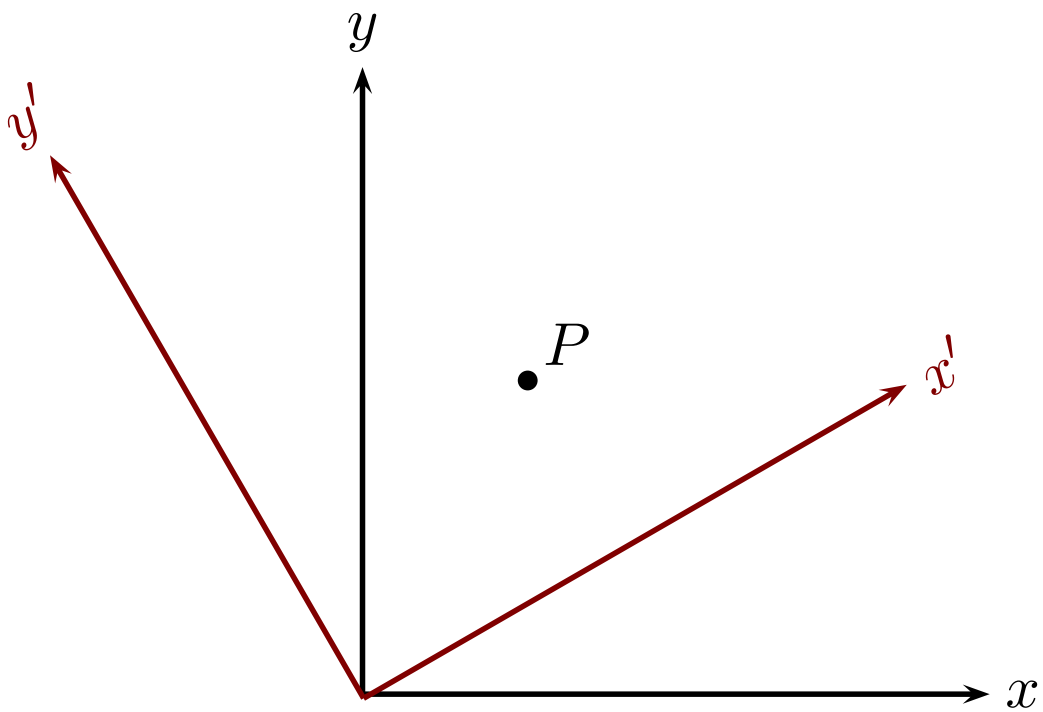


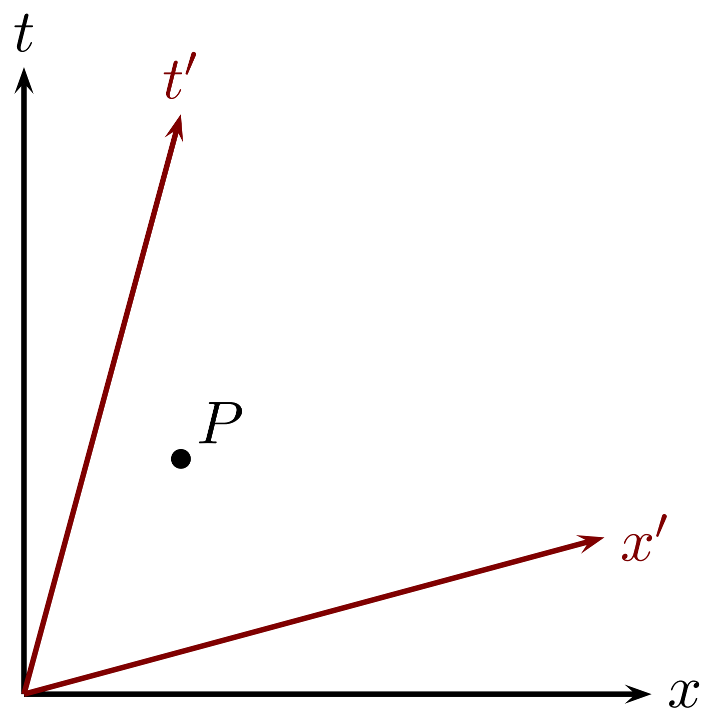
Particle Collisions



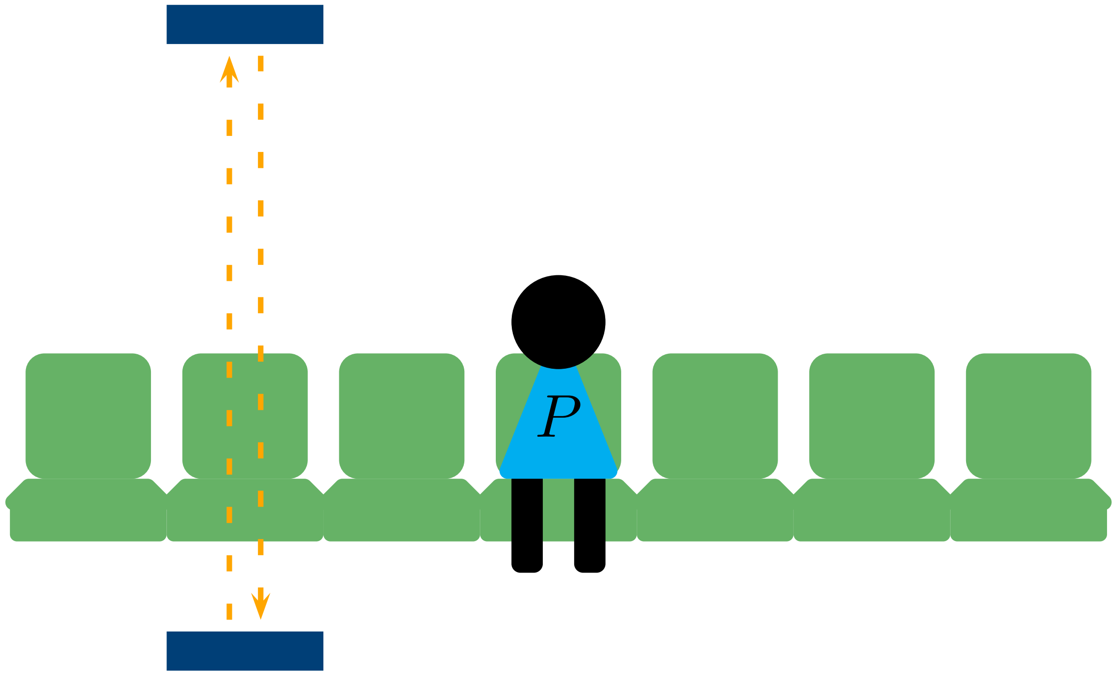
$$\sum_i \boldsymbol{p}_i = \sum_o \boldsymbol{p}_o$$
$$\sum_i \boldsymbol{E}_i = \sum_o \boldsymbol{E}_o$$
with $E^2_{(i,o)} = p^2_{(i,o)}c^2 + m_{(i,o)}c^4$

Rotations and Lorentz Transforms

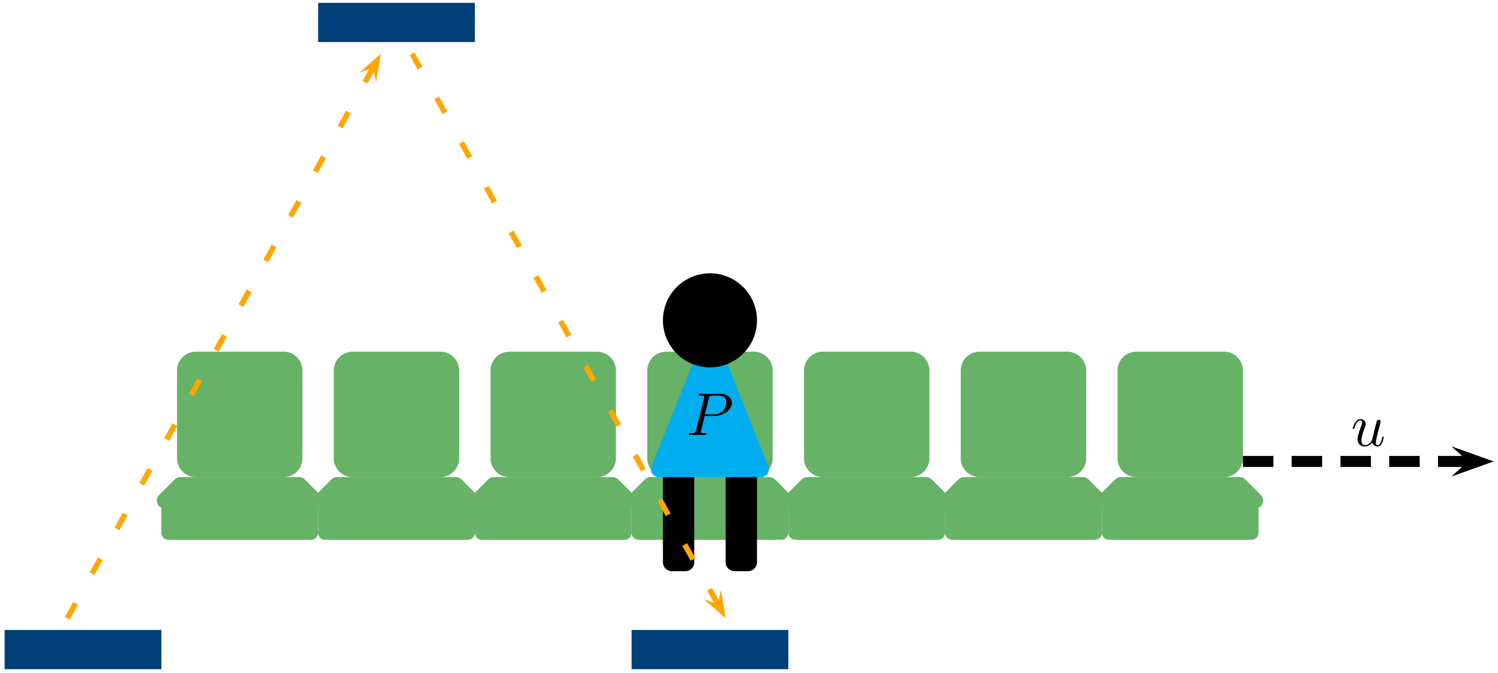




Clock on a Train

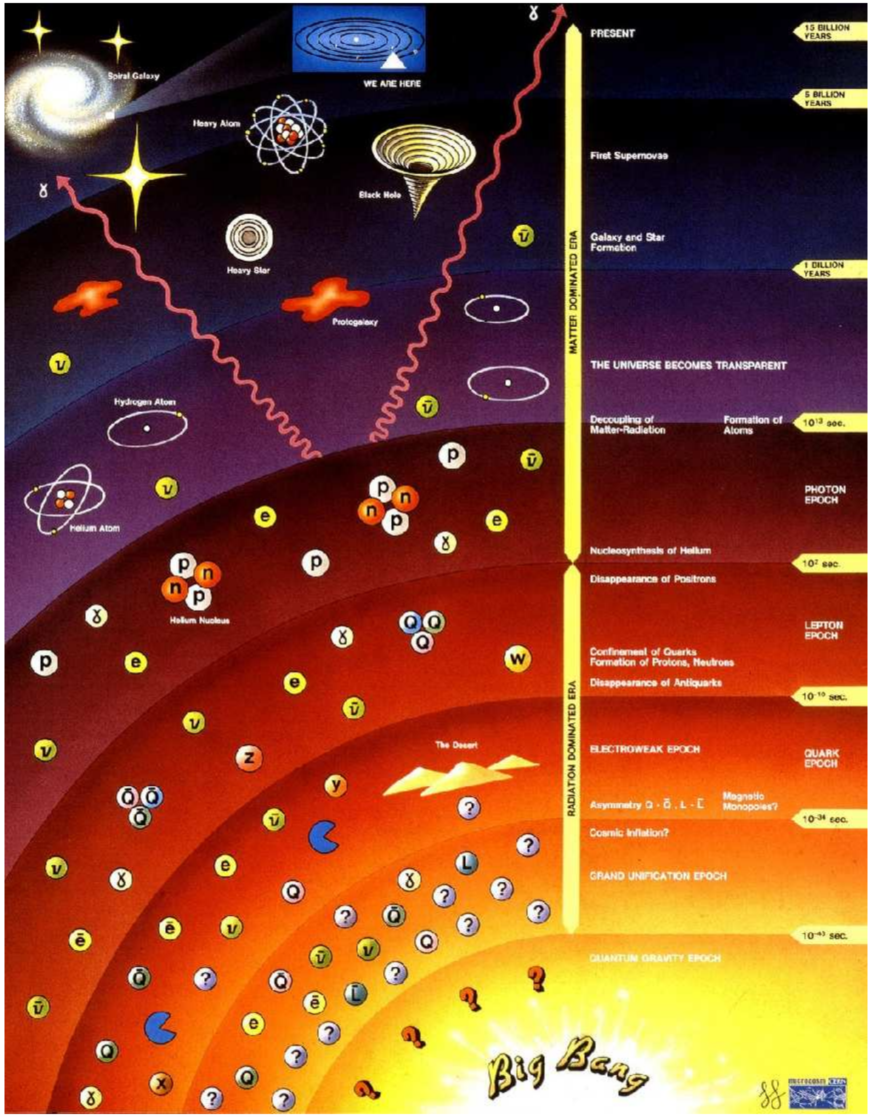


Clock on a Train



Introduction to Classwork

History of the Universe



Now ($t = 13.7 \cdot 10^9$ years) $T = 3\,\mathrm{K}$

Stars form ($t = 10^9$ years) $T = 15\,\mathrm{K}$

Atoms form ($t = 300,000$ years)

Nuclei form ($t = 100$ s) $T = 10^9\,\mathrm{K}$

Protons and neutrons form,
Antiquarks disappear (10^{-10} s)

Quarks form ($t = 10^{-34}$ s?)

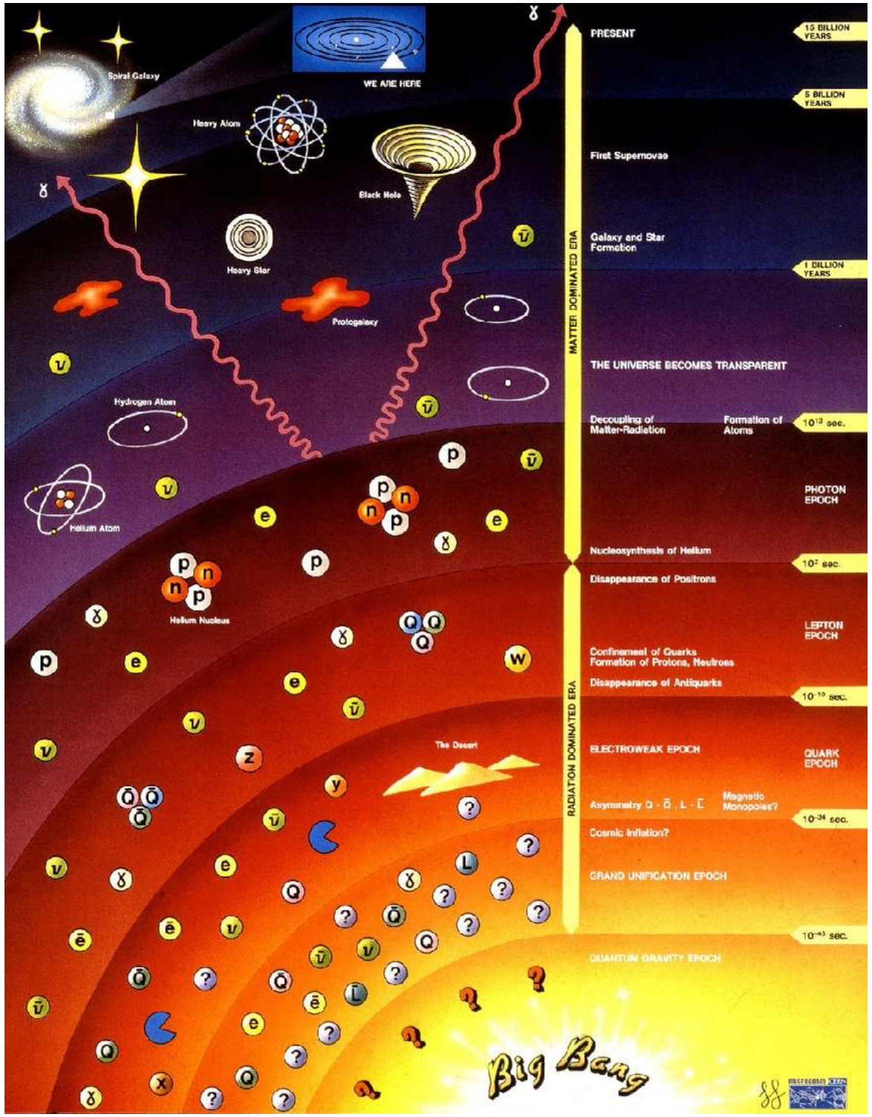
$T = 10^{28}\,\mathrm{K}$

????

Big Bang ($t = 0$)

Imperial College London

History of the Universe



At large temperatures the reaction below occurs:

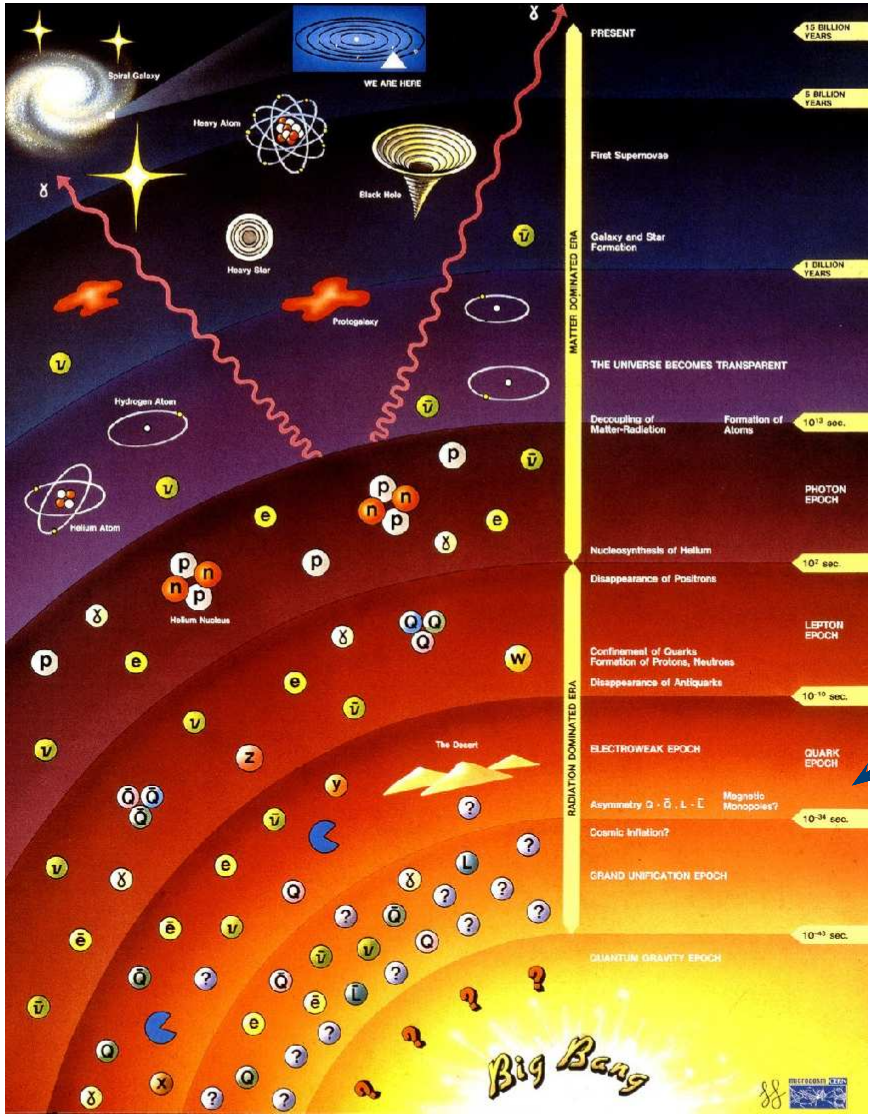
$$\gamma + \gamma \leftrightarrow \text{particle} + \text{antiparticle}$$

For a particle of mass m, the photons must have energy $E_\gamma > mc^2$.

As long as $E = kT > 2mc^2$ such pairs create and annihilate. Below this threshold the reaction stops.

Particle	Mass	T
W boson	$80\ \mathrm{GeV}/c^2$	$10^{15}\ \mathrm{K}$
d quark	$5\ \mathrm{MeV}/c^2$	$10^{11}\ \mathrm{K}$
electron	$511\ \mathrm{keV}/c^2$	$10^{10}\ \mathrm{K}$

History of the Universe

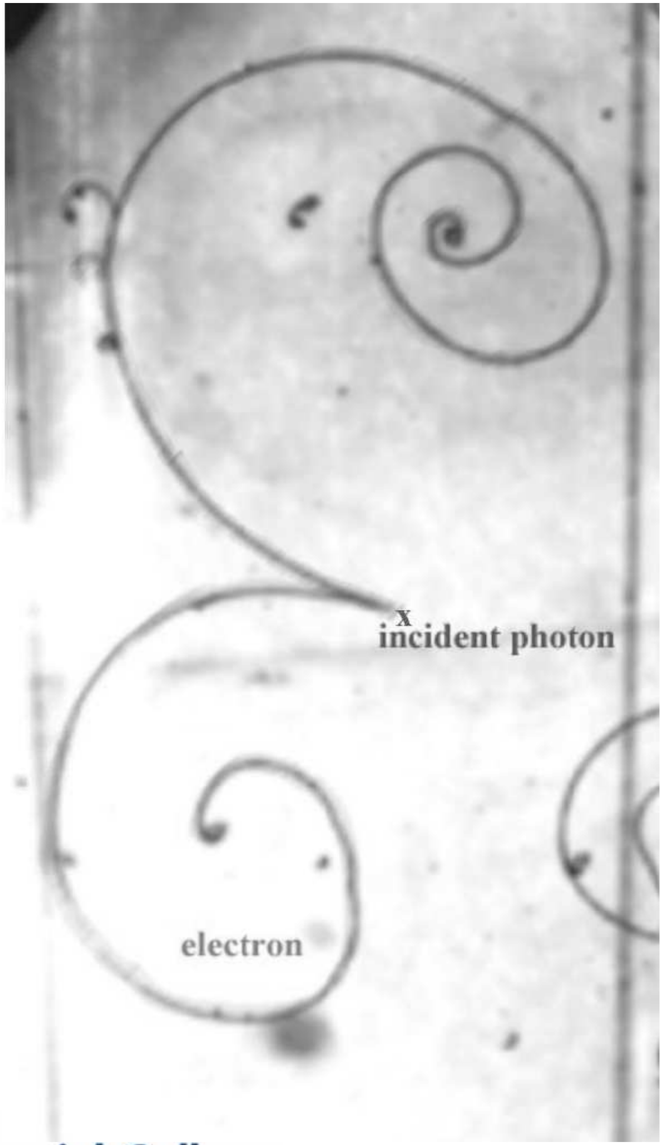


In collider experiments we recreate the conditions of the primordial Universe according to the available energy.

Accelerator	Energy	T
LEP	$200\,\mathrm{GeV}$	$10^{15}\,\mathrm{K}$
Tevatron	$2\,\mathrm{TeV}$	$10^{16}\,\mathrm{K}$
LHC	$14\,\mathrm{TeV}$	$10^{17}\,\mathrm{K}$

Particle	Mass	T
W boson	$80\,\mathrm{GeV}/c^2$	$10^{15}\,\mathrm{K}$
d quark	$5\,\mathrm{MeV}/c^2$	$10^{11}\,\mathrm{K}$
electron	$511\,\mathrm{keV}/c^2$	$10^{10}\,\mathrm{K}$

There's something wrong



- This bubble chamber picture shows a pair creation

$$\gamma + \gamma \rightarrow e^{+} + e^{-}$$

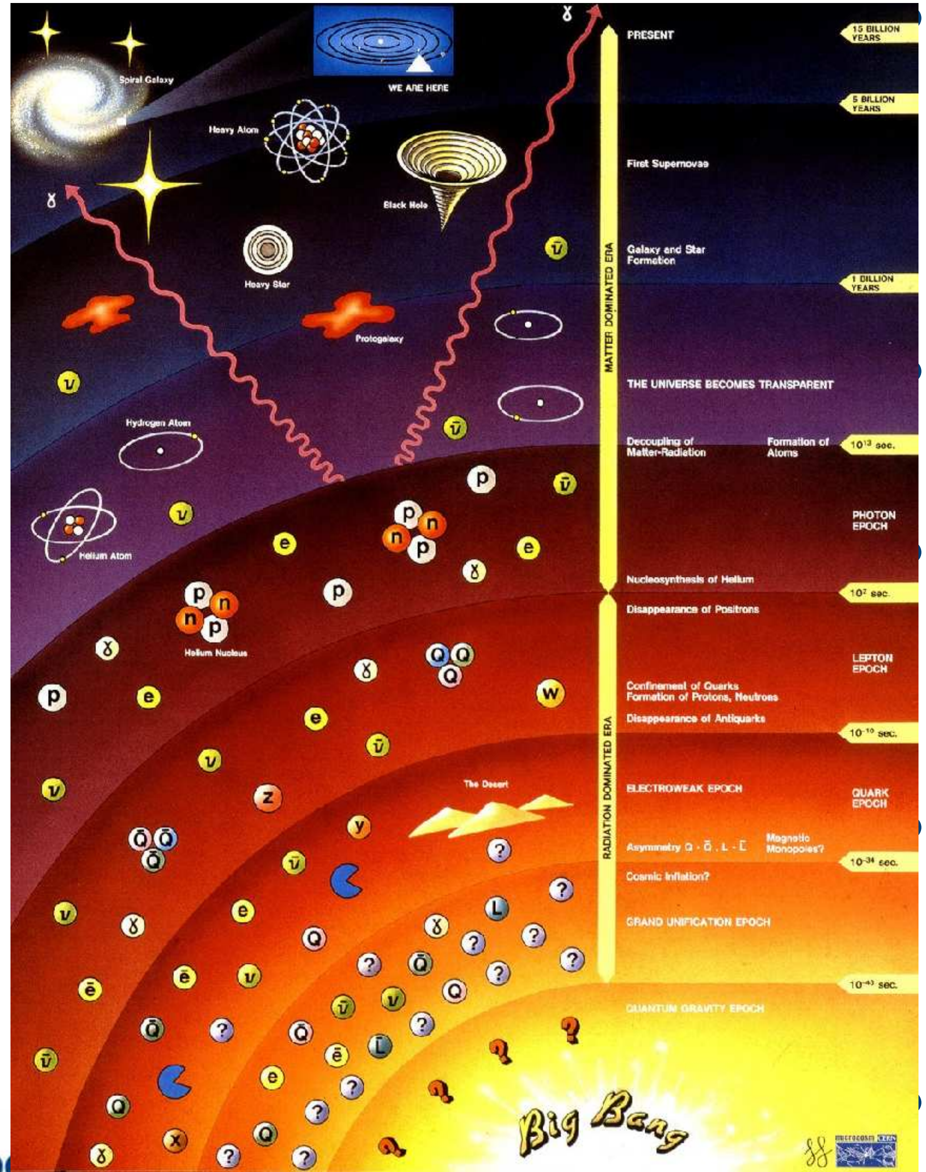
- Particle and antiparticles are always created in pairs
- And they annihilate by pairs

$$e^{+} + e^{-} \rightarrow \gamma + \gamma$$

- Hence:

$$\text{Particles} - \text{Antiparticles} = 0$$

There's something wrong



- This bubble chamber picture shows a pair creation

$$\gamma + \gamma \rightarrow e^+ + e^-$$

- Particle and antiparticles are always created in pairs
- And they annihilate by pairs

$$e^+ + e^- \rightarrow \gamma + \gamma$$

- Hence:

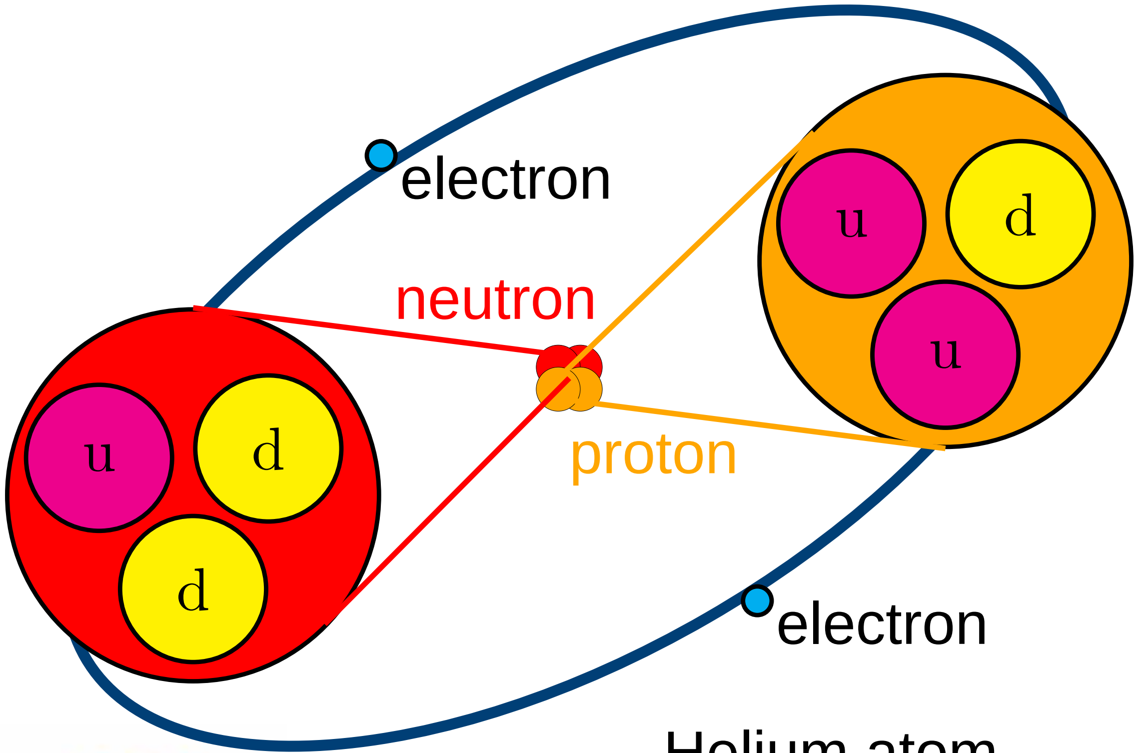
$$\text{Particles} - \text{Antiparticles} = 0$$

- Where have all antiparticles gone?

Antimatter

Antimatter factory:

1. Replace electrons by positrons
2. Replace all quarks by antiquarks

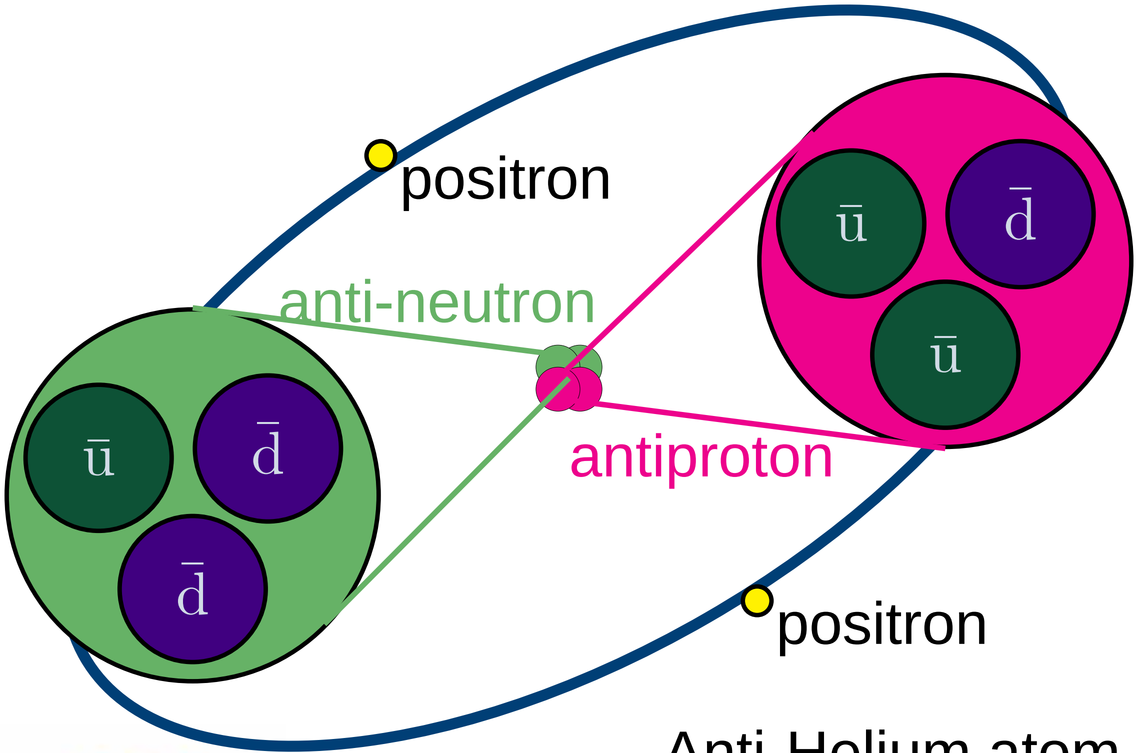


Helium atom

Antimatter

Antimatter factory:

1. Replace electrons by positrons
2. Replace all quarks by antiquarks



Anti-Helium atom

CERN has an anti-atom production facility.

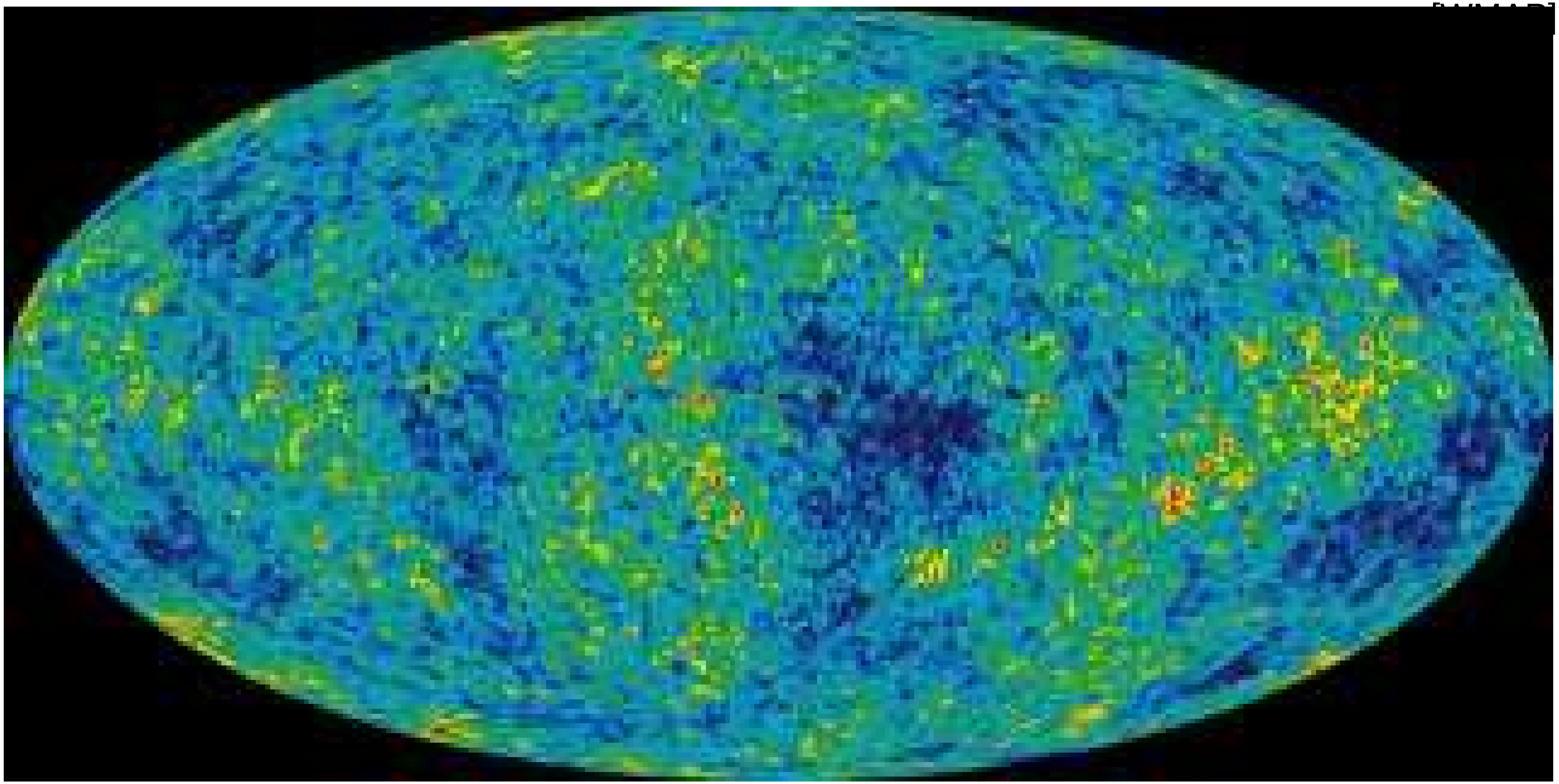
- Produced thousands of anti-hydrogen atoms
- Survive for milliseconds

$\rightarrow$ Still allows to study them

Does antimatter have the same properties as matter?

Antimatter in the Universe

[WMAP]



There should be annihilation photons — can’t see any.

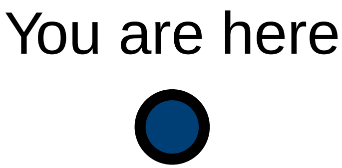
If there was a tiny difference

$10^{10} + 1$ quarks

10^{10} antiquarks

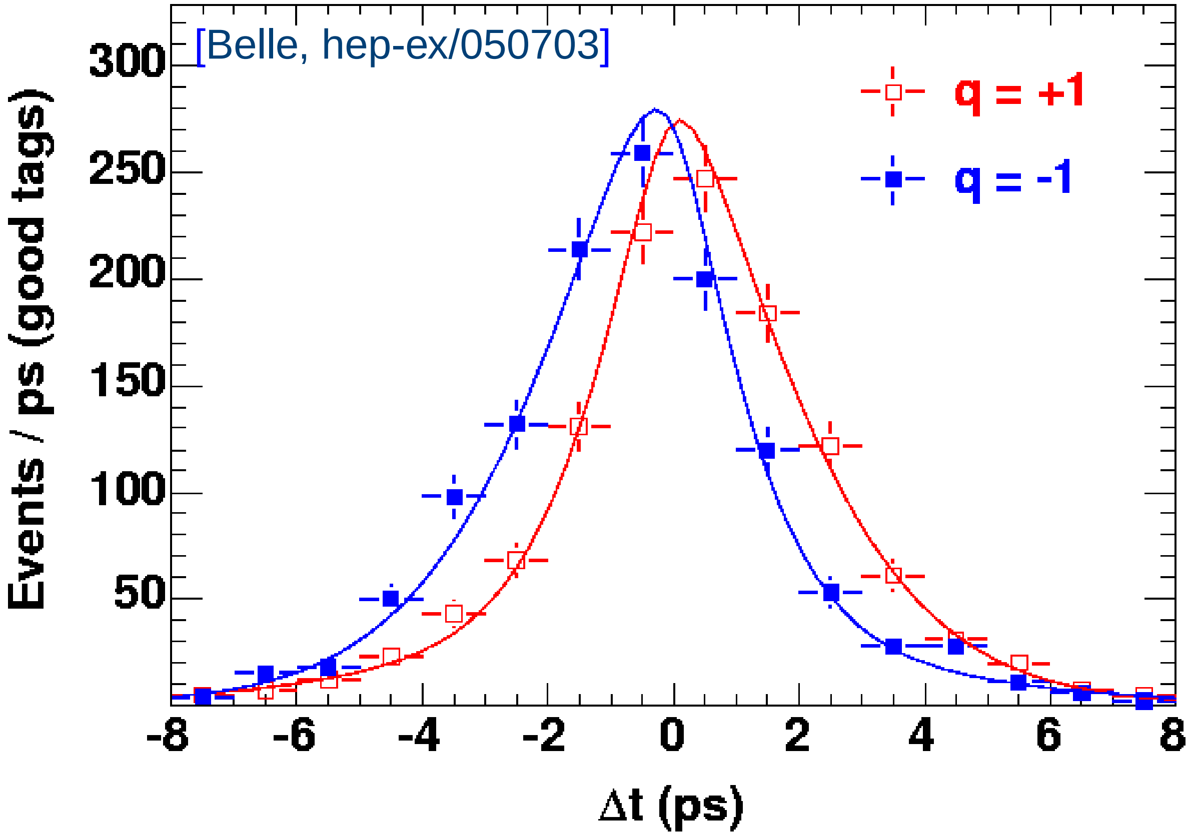
- If some mechanism slightly favoured matter of antimatter it could explain why there's any matter left and not just photons.
- Such a mechanism exists: CP violation

If there was a tiny difference



- If some mechanism slightly favoured matter of antimatter it could explain why there's any matter left and not just photons.
- Such a mechanism exists: CP violation

CP violation exists



The strongest matter-antimatter asymmetry ever measured: Number of $\mathrm{b} \to \mathrm{c\bar{c}s}$ and $\mathrm{\bar{b}} \to \mathrm{\bar{c}c\bar{s}}$ decays versus time.

Yet it's not enough: The asymmetry is a factor 10^{10} too small!

How to produce flying B mesons?

1. Collide and annihilate e^- & e^+
2. If the energy is right it will produce a particle called $\Upsilon(4\mathrm{S})$
3. The $\Upsilon(4\mathrm{S})$ immediately decays to two B mesons
   - But the B mesons will hardly move before they decay

   ➜ Need to give them a kick
4. If the e^- and e^+ have different energies, the $\Upsilon(4\mathrm{S})$ will have momentum and the B mesons will fly