November 22, 2007

Relativity Classwork 1

The Perils Of Paul

This problem is an old one; the Pole and Barn "paradox". In fact some of you had already worried about potentially absurd conclusions resulting from contraction along the direction of relative motion. Hopefully, after working through this classwork, you will have convinced yourselves that there is in fact neither paradox nor absurdity.

Paul Volta, the champion pole-vaulter, represents the Blackett Lab at the Imperial games. He has a pole 20 m long and his run-up is so fast that his pole appears contracted to a length of only $L = 10\,\mathrm{m}$. To achieve such a speed he needs a long run-up and he has to pass through the changing rooms, which are just over $L = 10\,\mathrm{m}$ long. In an attempt to remove him from the competition, students from the engineering faculty install a photocell system, designed to close both entrance and exit doors at the instant when the *front* end of his pole reaches the *exit* door.....with obvious disastrous consequences.

Fortunately the physicists hear of this dastardly plot. Though they cannot disconnect the device or prevent the doors closing, those who have done the electronics section in the laboratory devise another system which rapidly reopens the doors allowing Paul a free run.

Paul is told all this and is quite relaxed, until he begins his run-up towards the changing room, when he is dismayed to observe it has contracted. Too busy training to have attended the first year relativity course he worries about losing his title, his reputation and 15 metres of pole.

1. From the fact that the pole appears contracted to $10\,\mathrm{m}$ calculate the value of γ. Hence calculate Paul's speed, u.

2. Calculate the length of the changing room according to Paul.

Take as origin of coordinates and time, the event O, where the *front* of the pole reaches the *entrance* door of the changing room. This event is shown in the figure, for the sports ground frame above and for Paul's frame below.

3. Write down expressions (in terms of L, the length of the changing rooms in the sports ground frame, and u) for the space-time coordinates in the sports ground frame (S) of the following events:

   F: The *front* end of Paul's pole reaches the *exit* door.

   B: The *back* end of Paul's pole reaches the *entrance* door.

4. Use the Lorentz Transformation equations to obtain expressions for space-time coordinates (in terms of L and u) of these same events in Paul's frame (P).

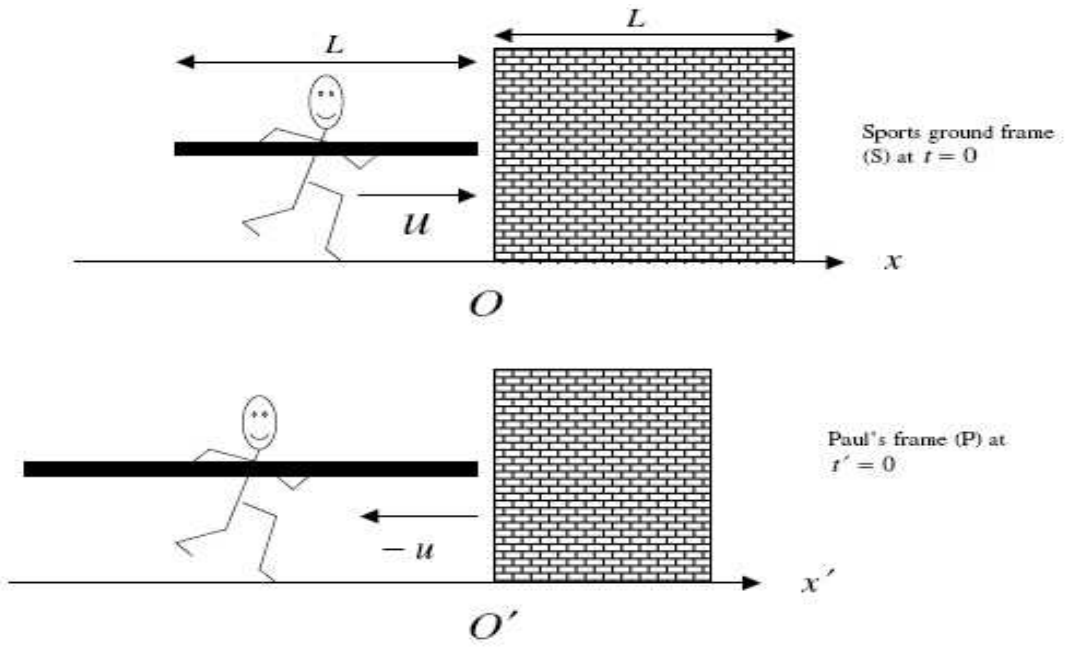

Figure 1: Paul in the Barn.

5. Calculate the times t_F, t_B, t'_F and t'_B *in nanoseconds*. (t' refers to time as measured in Paul's Frame).

6. Compare the values for t'_F and t'_B, obtained from qu.5. What can you say about Paul's chances ?

If time permits

On a graph (space-time diagram) of t vs x (with the time axis vertical) one for each frame, plot the trajectories of the entrance and exit doors, the front and back of the pole and the coordinates of the events O, F and B.

Answers to numerical questions:

1. 2, $2.6 \times 10^8\,\mathrm{m}\,s^{-1}$.

2. $5\,\mathrm{m}$.

5. 38.5, 38.5, 19.2, and $77.0\,\mathrm{ns}$.

6. Good.