Tuesday 17th May 2005

Quantum Physics Classwork 5

The Quantum Mechanics of Bricks

It is often assumed that quantum effects are unimportant in the everyday world. In fact, however, very few of the properties of everyday objects can be understood without invoking quantum mechanics. In this classwork, you will see how quantum mechanics determines the density of a brick.

1. Bricks are made of clay, the main ingredients of which are SiO_2 and Al_2O_3. The average mass of an atom in either of these materials is about 20 atomic mass units. Assuming that the average atomic radius is about 1 Å, estimate the density of a brick. (Real bricks are several times lighter than this estimate suggests, presumably because they are porous.)

But what determines the size of an atom? (According to classical physics, accelerating charges such as orbiting electrons should emit electromagnetic radiation and spiral inexorably into the nucleus.) The rest of this classwork shows how to estimate the size of an atom using Heisenberg's uncertainty principle,

$$\triangle x \triangle p_x \geq \hbar/2 \ ,$$

where $\triangle x$ is the root mean square uncertainty in the x component of the position vector and $\triangle p_x$ is the root mean square uncertainty in the x component of the momentum vector.

2. By definition, $(\triangle x)^2$ is the average of $(x - \langle x \rangle)^2$ and $(\triangle p_x)^2$ is the average of $(p_x - \langle p_x \rangle)^2$. If the atom is stationary and sitting at the origin, what are the values of $\langle x \rangle$ and $\langle p_x \rangle$?

3. Given that the root mean square radius $\sqrt{\langle r^2 \rangle}$ of the atom is a and that the atom is spherical, show that $\triangle x = a/\sqrt{3}$.

4. Assuming that the equality holds in the uncertainty principle, obtain expressions for the average values of p_x^2 and $\frac{1}{2}mv_x^2$ of the orbiting electron.

5. The potential energy of the outermost electron in the atom is approximately
$$V = -\frac{e^2}{4\pi\varepsilon_0 a} \ .$$
Why is the atomic number Z, which determines the nuclear charge, absent from this expression?

6. Write down the total energy ($\mathrm{KE}_x + \mathrm{KE}_y + \mathrm{KE}_z + \mathrm{PE}$) of the outermost electron and differentiate to find the value of a at which the total energy is minimised. Hence estimate the radius of the orbit (in Å) and the ionisation energy (in eV) of the atom. (The ionisation energy is the energy required to strip off the outermost electron.)

   Are these estimates reasonable?

7. Explain in words how the size of an atom is determined by the balance between potential energy and kinetic energy terms. Which contribution favours small/large atoms? How would the balance, and hence the atomic size, alter if the electrons were replaced by heavier particles such as muons? The sizes of atoms depend only weakly on atomic number. Why?

Useful Constants

Atomic mass unit	m_u	=	1.66×10^{-27} kg
Elementary charge	e	=	1.60×10^{-19} C
Mass of electron	m_e	=	9.11×10^{-31} kg
Permittivity of vacuum	ε_0	=	8.85×10^{-12} F m^{-1}
Planck's constant	h	=	6.63×10^{-34} J s
Planck's constant/2π	$\hbar$	=	1.05×10^{-34} J s