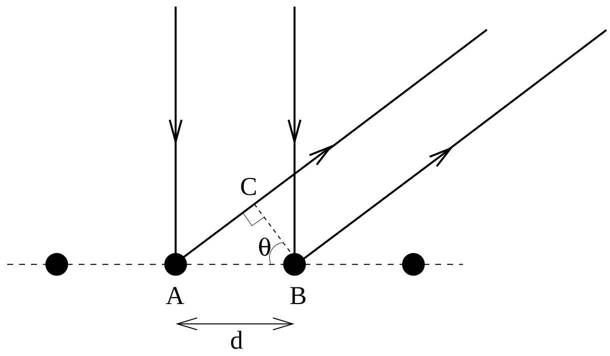
Monday 7th March 2005

Answer to Quantum Physics Classwork 3

Electron Diffraction

1. The path difference between waves scattered from atoms A and B is AC, where BC is perpendicular to AC and $\widehat{\mathrm{ABC}} = \theta$.



   From the diagram, the path difference $AC = d \sin\theta$. Constructive interference occurs when AC is a whole number of wavelengths, and hence when

$$d \sin\theta = n\lambda \qquad\qquad (n = 1, 2, 3, \ldots).$$

2. For $n = 1$, $\lambda = d\sin\theta = (0.215 \times 10^{-9}) \sin 50^o$. Hence, the wavelength is 0.165 nm.

3. One eV is the energy gained by an electron in falling through a potential difference of 1 volt. Hence, if $V_0 = 54$ volts, the kinetic energy K of the electrons is 54 eV.

4. $1\,\mathrm{eV} = e$ Joules. Hence,

$$K = \frac{1}{2} m_e v^2 = 54\,\mathrm{eV} = 54e \text{ Joules},$$

   and

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2 \times 54 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} = 4.36 \times 10^6\,\mathrm{ms}^{-1}\ .$$

This is $\sim 1\%$ of the speed of light — not relativistic. We could also have deduced this by noting that the KE of 54 eV is $\ll$ the rest mass energy $m_e c^2 = 0.511 \times 10^6$ eV.

5. Kinetic energy $K = p^2/2m_e$ and hence $p = \sqrt{2m_e K}$. Since, as in (4), $K = eV_0$ Joules, the de Broglie equation predicts

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV_0}} \ .$$

6. If the potential difference is 54 volts,

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times (9.11 \times 10^{-31}) \times (1.60 \times 10^{-19}) \times 54}} = 0.167\,\mathrm{nm} \ .$$

This agrees with the answer in (2) to within experimental errors.

7. The electron KE of 600 eV corresponds to an accelerating voltage V_0 of 600 volts. Since

$$\lambda \propto \frac{1}{\sqrt{V_0}}$$

we get

$$\lambda_{\mathrm{T}} = \lambda_{\mathrm{DG}} \sqrt{\frac{V_{\mathrm{DG}}}{V_{\mathrm{T}}}} = 0.167 \sqrt{\frac{54}{600}}\ \mathrm{nm} \ .$$

Therefore, the wavelength of Thomson's electrons was 0.050 nm.

8. G.P. Thomson was a Professor of Physics at Imperial College and Head of Department from 1930–52.