Monday 28th February 2005

Quantum Physics Classwork 2

Photon Energy and Momentum

You already know that the energy E and momentum p of a photon of frequency f and wavelength λ are given by

$$E = hf \quad \text{(Planck's Equation)} \tag{1}$$

$$p = h/\lambda \quad \text{(De Broglie's Equation)} \tag{2}$$

where $h = 6.63 \times 10^{-34}$ Js is Planck's constant. In this classwork, you will investigate some of the consequences of Eqs. (1) and (2).

1. (a) Rewrite Eqs. (1) and (2) in terms of the angular frequency ω, the wavevector k and the constant $\hbar \equiv h/2\pi$.

   (b) Express the speed of light c in terms of: (i) f and λ; and (ii) ω and k.

   (c) Show that Eqs. (1) and (2) are consistent with the relation $E = pc$ expected for massless particles.

2. An electron and a positron (which has the same mass as an electron but the opposite charge) annihilate at rest to produce two photons.

   (a) The two photons have the same wavelength and travel in opposite directions. Why?

   (b) Calculate the photon momentum and wavelength.

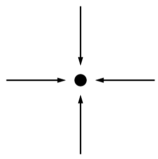
   ($m_e = 511\,\text{keV}/c^2$; $e = 1.6 \times 10^{-19}$ C; $c = 3.00 \times 10^8\,\text{ms}^{-1}$.)

3. The Nobel prize in physics for the year 2001 was awarded to the first groups ever to create a "Bose condensate" (don't worry what this means) of atoms in a trap. To accomplish this, the atoms had to be cooled to a few hundred nanokelvin.

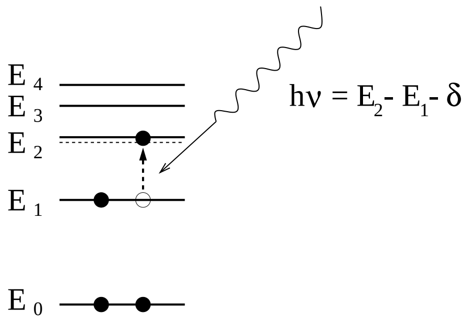
   (a) According to the principle of equipartition, the average kinetic energy of an atom at temperature T is $3k_BT/2$. What is the average speed of a ^{87}Rb atom (the first atom for which Bose condensation was achieved) at 100 nK? ($k_B = 1.38 \times 10^{-23}\,\text{JK}^{-1}$; 1 atomic mass unit $= 1.66 \times 10^{-27}$ kg.)

(b) The penultimate stage of the cooling (slowing down) process was carried out using the "optical molasses" technique. The developers of this and other laser cooling methods won Nobel prizes in 1997.

An optical molasses is created by shining lasers at the atoms from all sides.



The laser frequencies are tuned to lie just below the energy of an atomic transition. Because the energy of the photons hf does not quite match the energy of the transition $\Delta E = E_2 - E_1$, the photons cannot be absorbed efficiently.



Now suppose that the atom starts moving towards one of lasers. As soon as it does so, the photons arriving from that laser are very slightly blue shifted by the Doppler effect. This increases their frequency and brings their energy up to the value required to excite the atom from energy level E_1 to energy level E_2. Photons from the laser towards which the atom is moving are thus absorbed more efficiently than photons from the other lasers. Every photon absorbed by the atom is later reradiated, but in a random direction.

How does this cool the atom? How much does the speed of a ^{87}Rb atom change when it absorbs a single photon of wavelength 550 nm? Could the optical molasses technique be used to cool to 100 nK?