Answer to Quantum Physics Classwork 2

Friday 29th February 2008

Photon Energy and Momentum

1. (a) $E = \hbar\omega$; $p = \hbar k$.

   (b) (i) $c = f\lambda$; (ii) $c = \omega/k$.

   (c) Since $c = f\lambda$, Eq. (1) may be rewritten $E = hc/\lambda$. Replacing h/λ by p using Eq. (2) gives $E = pc$.

2. (a) When the electron and positron annihilate, both momentum and energy must be conserved. Since the electron and positron are initially at rest, the total momentum is equal to zero and must remain equal to zero. This explains why the two photons have equal and opposite momentum. Since the energy and (magnitude of) momentum of a photon are related via $E = pc$, photons with the same (magnitude of) momentum must also have the same energy.

   (b) The photon energy can be obtained by conservation of energy:

$$2mc^2 \;=\; 2 \times 511\,\text{keV} \;=\; 2E_{\text{photon}}\,,$$

   and hence $E_{\text{photon}} \approx 511\,\text{keV}$. The photon momentum is

$$p_{\text{photon}} \;=\; E_{\text{photon}}/c \;\approx\; 511\,\text{keV}/c \;\approx\; \frac{511 \times 10^3 \times 1.6 \times 10^{-19}}{3.00 \times 10^8} \;\approx\; 2.73 \times 10^{-22}\,\text{kg}\,\text{ms}^{-1}\,,$$

   and the photon wavelength is

$$\lambda \;=\; \frac{h}{p} \;\approx\; \frac{6.63 \times 10^{-34}}{2.73 \times 10^{-22}} \;\approx\; 2.43 \times 10^{-12}\,\text{m}\,.$$

3. (a) $$\frac{1}{2}mv^2 = 3k_BT/2 \quad\Rightarrow\quad v = \sqrt{\frac{3k_BT}{m}} \;\approx\; 5.35 \times 10^{-3}\,\text{ms}^{-1}\,.$$

   (b) As soon as an atom starts moving away from the region where the beams cross, it starts absorbing photons from the laser it is heading towards. For every photon absorbed, the atom receives a momentum kick back towards the centre. These kicks prevent the atom from building up any appreciable speed away from the centre. As we saw in (a), slow atoms are cold atoms. (The kicks from the re-radiated photons also change the velocity of the atom, but since these are in random directions they do not have any effect on the average velocity.)

Whenever the atom absorbs a photon, it receives a momentum kick of $p = h/\lambda$. Hence

$$\triangle p = m\triangle v = h/\lambda ,$$

from which we obtain,

$$\triangle v = \frac{h}{\lambda m} = 8.35 \times 10^{-3}\,\mathrm{ms}^{-1} .$$

This looks to be of the right order of magnitude to cool the atoms to a few hundred nK, but since each kick changes the speed by more than the average speed at 100 nK, and since there are many kicks, it is not good enough (in fact, the minimum attainable temperature is a few hundred μK). The original Bose-Einstein condensation experiments used another technique, called evaporative cooling, for the final step. The strength of the trap was gradually reduced, allowing the atoms with the most energy (the fastest ones) to escape. Eventually, only the slowest and coldest atoms were left behind.