Quantum Physics

Handout: Black Body Radiation (BBR)

Background

Planck, in 1900, was attempting to understand the spectrum of BBR, and introduced the idea of quantisation in the exchange of energy between the radiation field and the inside walls of a Black-Body (BB).

What is a spectrum? The "spectrum" or "spectral distribution" of radiation is the distribution of intensity emitted by a source, or absorbed by some material, as a function of the wavelength, λ, or equivalently the frequency, ν, of the e.m. energy. It tells us what is the energy being emitted or absorbed at different wavelengths by a material.

What is a BB? BBR is also known as cavity radiation. The idea is that a cavity and the radiation field inside it are allowed to come to equilibrium, in which case the radiative properties of the cavity can be accurately described (in principle). This is achieved by means of an isolated cavity, with high emissivity (ie black) inside walls, and a very small aperture to the outside world. Thus, the photons inside will experience many interactions with the walls, and equilibrium will be established at the temperature of the interior.

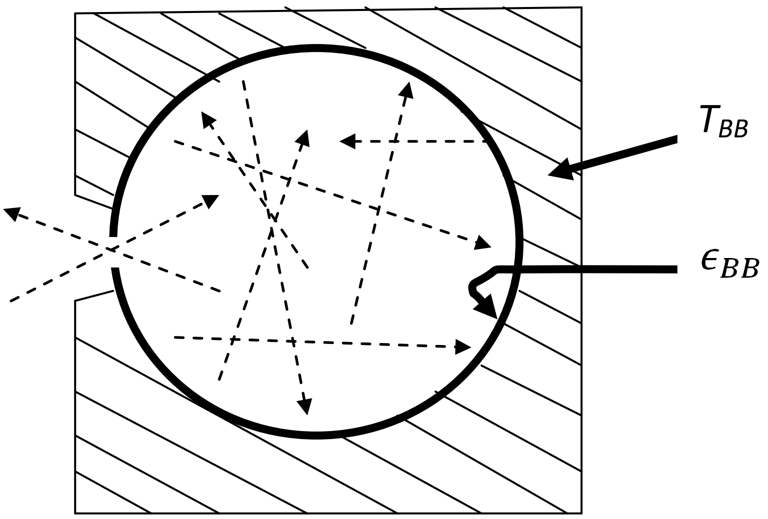

Fig BBR.1 A Black-Body cavity

Another analogue is an oven with a small hole in the wall. The idea of the "brightness temperature", T_B , of such an oven is familiar: the colour (or peak wavelength) of the interior, seen through the hole, is a measure of the temperature of the oven (or the BB).

A BB can in principle provide a well-understood, accurate radiation source, capable of being used as an international standard of radiative intensity, and is used as such in all national standard laboratories, such as NPL in the UK and NIST in the USA. BBs are also used extensively in experiments as calibration sources: for example, in a space project being led by our group here at Imperial, called the Geostationary Earth Radiation Budget experiment GERB), a BB cavity is carried on board the instrument, on the satellite, so that incoming radiative energy from the Earth, across a wide spectral range, can be compared with the known energy coming from the BB, and so calibrated in absolute terms. The on-board BB is calibrated before launch against a standard BB source, provided to us by NPL. The aim of the GERB experiment is to observe, very precisely, the variations of the Earth's radiative energy balance, on time scales from 15 minutes to decades. Such observation are vital to our understanding of the variability of the climate system. Go to the GERB project web sites, starting at:

http://www3.imperial.ac.uk/spat/research/missions/atmos_missions/gerb

Planck's work

Existing classical theory due to Rayleigh was based on the idea of a number of 'normal modes' of e.m. wave energy each being given an energy of $k_B T$. This led to something rather melodramatically known as "THE ULTRA-VIOLET CATASTROPHE", with the intensity of radiation from such a cavity found to have a $1/\lambda^4$ dependence, as in:

$$I(\lambda) = \frac{2\pi c k_B T}{\lambda^4} \qquad \text{[BBR1]}$$

and so shooting off to infinity as $\lambda \rightarrow 0$. This meant, of course, that the total energy over all wavelengths, $\int_0^\infty I(\lambda)d\lambda$, also tended to ∞. Experimentally, this λ^{-4}dependence was not observed, although eq. [BBR1] was close to the observed in the long wave limit.

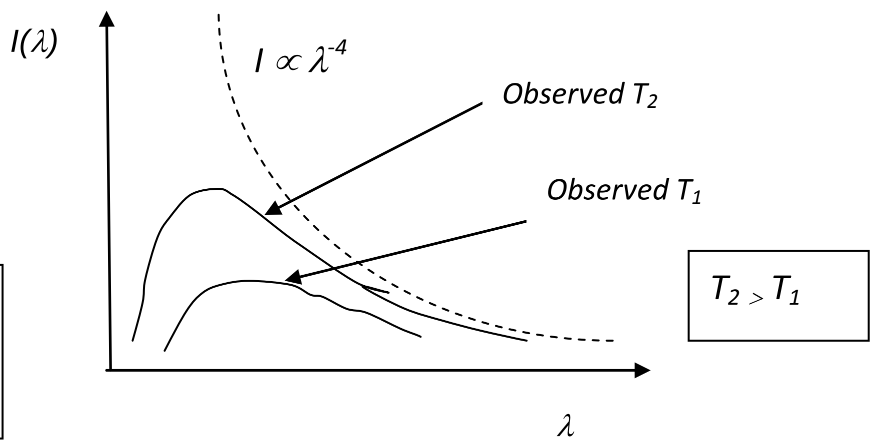

Fig BBR.2 Black Body radiation spectrum

Observations showed that the BBR curve turned over at short wavelengths, and tended towards zero at zero wavelength, a much more realistic result, that avoided the UV catastrophe It was also observed that the peak of the BBR curve shifted to shorter wavelengths, and higher intensities as the temperature rose.

Planck made what was at the time an unusual assumption, which led to a precise fit of theory and observation. He postulated that at any particular frequency, ν , energy in the e.m. radiation field in the cavity exchanged with the walls only in a quantised way, at certain values of frequency,

$$E = nh\nu, \qquad \text{[BBR2]}$$

where n is an integer (0, 1, 2, 3….) and h is a universal constant,

h = Planck's constant ≈ $6.63.10^{-34}$ J s.

The derivation of the BBR law that Planck obtained is not examinable, because it requires elements of statistical mechanics that you will not study until year 2. Nevertheless, it is reproduced here for completeness. The Planck derivation started from the calculation (taken from statistical physics: 2nd year course) of the number of possible energy states that the photons (which behave as Fermionic particles[1]) can take.

In frequency terms, the average number of photons per energy state for Fermionic particles such as the photon is given by[1] the standard expression

$$f(\nu) = \frac{1}{\left[exp\left(\frac{h\nu}{k_B T}\right) - 1\right]} \qquad \text{[BBR3]}$$

where the symbols all have their usual meaning. Statistical physics can also be used to show that for a simple box model, the number of energy states between frequencies ν and $\nu + d\nu$ is

$$g(\nu)d\nu = \frac{V 8\pi\nu^2 d\nu}{c^3} \qquad \text{[BBR4]}$$

where V is the volume of the 'box'.

So, the energy density within the cavity between frequencies ν and $\nu + d\nu$ is

$$u(\nu)d\nu = \frac{1}{V}.h\nu.f(\nu).g(\nu)d\nu, \qquad \text{[BBR5]}$$

since the energy per photon is $h\nu$. So,

[1] See: D H Trevena, "Statistical mechanics", Ellis Horwood, 1993, pp 90-92.

$$u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3} . \frac{d\nu}{\left[exp\left(\frac{h\nu}{k_B T}\right)-1\right]} . \qquad \text{[BBR6]}$$

Note that we can express this result as a function of wavelength instead of frequency, by converting, from frequency to wavelength dependence, noting that $c = \nu\lambda$ and that d$\nu = \frac{-c}{\lambda^2} d\lambda$, so that

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left[exp\left(\frac{hc}{\lambda k_B T}\right)-1\right]} \qquad \text{[BBR7]}$$

u is the energy density in the cavity, but a more useful parameter is often the intensity of energy passing normally through some unit area, such as the aperture in Fig BBR1, for example. It can be shown[2] that in order to convert from energy density to the flux passing an area A, we must multiply by $\frac{c}{4}$, the *c* in order to calculate the flux of energy passing through the area (at the speed of light) per second, and the ¼ being a geometrical factor, allowing for the full range of oblique incident angles that individual rays take as they pass through the area. Then, to convert from energy flux to energy intensity (or radiance), assuming the BB radiation field is isotropic, we must divide by a factor of π. In all, we multiply by a factor

$$\frac{c}{4\pi}$$

in order to convert from energy density within the cavity, $u(\nu)$, to energy intensity (radiance) per unit area, per unit solid angle, per unit frequency, at the aperture, $I(\nu)$. Making this adjustment to eq [BBR6], the intensity of energy obtained from a BB at a temperature, *T*, is

$$I(\nu) = \frac{c}{4\pi} . \frac{8\pi h\nu^3}{c^3} . \frac{1}{\left[exp\left(\frac{h\nu}{k_B T}\right)-1\right]} = \frac{2h\nu^3}{c^2\left[exp\left(\frac{h\nu}{k_B T}\right)-1\right]} \qquad \text{[BBR8]}$$

in units of J s^{-1} m^{-2} $ster^{-1}$ Hz^{-1} , which fitted the observed curve perfectly.

Another useful law is Stefan-Boltzmann's, which gives the total flux from a BB at a temperature, *T*, integrated over all frequencies, as follows.

The total energy per unit time emerging from a small aperture in the BB cavity is, using the above working,

2 See: R M Goody and Y L Yung, "Atmospheric Radiation", OUP, 1995, pp 16-19

$$F = \int_0^\infty \frac{c}{4} u(\nu) d\nu = \int_0^\infty \frac{c}{4} \frac{8\pi h\nu^3}{c^3} \cdot \frac{d\nu}{\left[exp\left(\frac{h\nu}{k_B T}\right) - 1\right]} \qquad \text{[BBR9]}$$

$$F = \frac{c}{4} \frac{8\pi h}{c^3} \left(\frac{k_B T}{h}\right)^4 \int_0^\infty \frac{x^3 dx}{e^x - 1} , \quad where \;\; x = {}^{h\nu}/_{k_B T}$$

and the definite integral on the r.h.s. takes the value ${}^{\pi^4}/_{15}$. This leads directly to the Stefan-Boltzmann law,

$$F = \sigma T^4, \qquad \text{[BBR10]}$$

where $\sigma = {}^{2\pi^5 k_B^4}/_{15c^2h^3}$ = 5.67 x 10^{-8} W $m^{-2}K^{-4}$ is the Stefan-Boltzmann constant .

Planck was awarded the 1918 Nobel Prize for his work.