MECHANICS PROBLEM SHEET 9, ANSWERS

1. $v = \sqrt{\dfrac{G M_{Earth}}{R_{Earth}}} = \left(\dfrac{6.67\times10^{-11}\times5.98\times10^{24}}{6.37\times10^{6}}\right)^{\frac{1}{2}}$

$= 7.91\times10^{3}\ ms^{-1}$

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2. $T = \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM}} = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$

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3. In geostationary orbit $T = 1\ day = 8.64\times10^{4}s$

Using Q2: $r = \left(\dfrac{T}{2\pi}\right)^{2/3}(GM)^{1/3}$

$= \left(\dfrac{8.64\times10^{4}}{2\pi}\right)^{2/3}\times\left(6.67\times10^{-11}\times5.98\times10^{24}\right)^{1/3}$

$= 4.22\times10^{7}m$

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4. Using Q2: $M = \dfrac{1}{G}\left(\dfrac{2\pi}{T}\right)^{2} r^{3}$

$= \dfrac{(2\pi)^{2}\times(5.84\times10^{8})^{3}}{6.67\times10^{-11}\times(1.16\times10^{6})^{2}} = 8.76\times10^{25}\ kg$

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5. (i) F_1 = force of Earth on moon $= \dfrac{G M_{moon} M_E}{R_1^{2}}$ Mass of Earth

where R_1 = Earth-Moon distance

F_2 = force of Sun on moon $= \dfrac{G M_{moon} M_S}{R_2^{2}}$ Mass of Sun

where R_2 = Sun-Moon distance $\simeq$ Sun-Earth distance

$$\therefore \frac{F_2}{F_1} = \frac{M_S}{M_E}\left(\frac{R_1}{R_2}\right)^2 = \frac{1.99\times10^{30}}{5.98\times10^{24}} \times \left(\frac{3.84\times10^{8}}{1.49\times10^{11}}\right)^2 = 2.21$$

(ii) The Moon DOES orbit the Sun. We can think of its orbit around the Earth as a small perturbation to its basic motion around the Sun

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6 (i) $K = \frac{1}{2} m_e v^2 \leftarrow v^2 = \frac{e^2}{4\pi\epsilon_0 m_e r}$ $\quad \therefore K = \frac{e^2}{8\pi\epsilon_0 r}$

$$E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

i.e. $|E| = |K| = \frac{1}{2}|U|$

(ii) $E_1 = \frac{-e^2}{8\pi\epsilon_0} \times \left(\frac{m_e e^2}{\hbar^2 4\pi\epsilon_0}\right) \leftarrow \frac{1}{r_1}$

$$= \frac{-m_e e^4}{32\pi^2\epsilon_0^2\hbar^2}$$

$$= -\frac{(9.11\times10^{-31})\times(1.60\times10^{-19})^4}{32\pi^2\times(8.85\times10^{-12}\times1.05\times10^{-34})^2} = -2.19\times10^{-18}\,\text{J}$$

(iii) $U(r=\infty)=0$ $\therefore$ minimum energy required to remove ground state electron to $r=\infty$ is $-E_1$
$\rightarrow$ ionization energy $= +2.19\times10^{-18}\,\text{J} \rightarrow 13.7\,\text{eV}$

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7 (i) $\frac{dU^*}{dr} = \frac{L^2}{2m}\left(\frac{-2}{r^3}\right) - GMm\left(\frac{-1}{r^2}\right)$

$= 0$ when $\frac{L^2}{mr^3} = \frac{GMm}{r^2}$

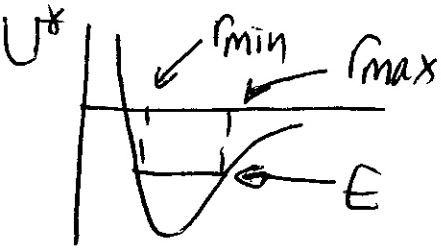
i.e. minimum is at $r = r_c = \frac{L^2}{GMm^2}$

At minimum: $\frac{L^2}{mr_c^2} = \frac{GMm}{r_c}$

$\therefore U^*_{min} = \frac{L^2}{2mr_c^2} - \frac{GMm}{r_c} = -\frac{GMm}{2r_c}$

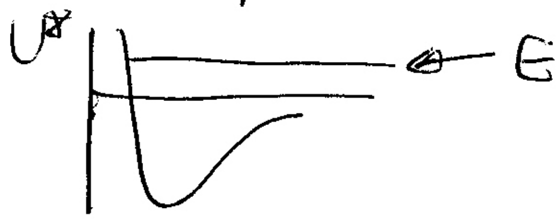
(ii) (a) $E = U^*_{min} \Rightarrow r = r_c = \text{const} \Rightarrow$ circular orbit

(b) $U^*_{min} < E < 0$



During orbit r changes from r_{min} (= perihelion) to r_{max} (= aphelion) $\Rightarrow$ elliptical orbit

(c) $E > 0$: object escapes to ∞



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8 (i) The composite body will not be torn apart if the gravitational attraction between the 2 spheres is greater than the tidal force

i.e. $\frac{GmM}{(2a)^2} > \frac{4GMma}{r^3}$

separation of centres of spheres $= 2a$

$\therefore r^3 > a^3 \, 16 \frac{M}{m}$ i.e. $r_{min} = a\left(\frac{16M}{m}\right)^{1/3}$

(ii) $M = \frac{4}{3}\pi R^3 \rho$ [R = radius of planet, ρ = density]

$$m = \frac{4}{3}\pi a^3 \rho$$

$$\therefore r_{min} = a\left(\frac{16R^3}{a^3}\right)^{1/3} \quad \therefore \frac{r_{min}}{R} = 16^{1/3} \simeq 2.52$$

(iii) Artificial satellites are held together by nuts & bolts, not gravity.
