# MECHANICS PROBLEM SHEET 7 ANSWERS

1. MoI of cd $= \frac{1}{2}M(b^2+a^2) = \frac{1}{2}\times 15\times10^{-3}\times\{(160\times10^{-3})^2+(7.5\times10^{-3})^2\}$

$$= 2.74\times10^{-5}\ \mathrm{kg\,m^2}$$

$T$ = period $= 60/400 = 0.15\,\mathrm{s}$ $\quad \omega = 2\pi/T = 41.9\ \mathrm{rad\,s^{-1}}$

$K = \frac{1}{2}I\omega^2 = 0.0261\,\mathrm{J}$, $\quad L = I\omega = 1.15\times10^{-3}\ \mathrm{kg\,m^2s^{-1}}$

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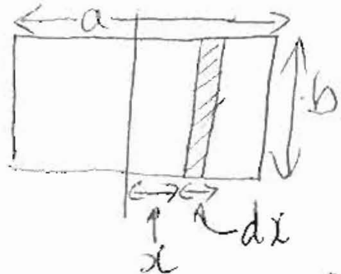
2 (i)




$M = \rho ab\Delta$ ← thickness

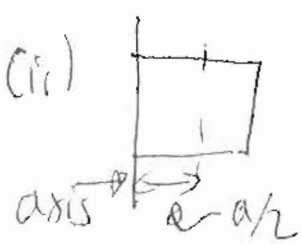
$dm$ = mass of element width $dx$ distance $x$ from axis $= \rho b\,dx\,\Delta$

$$dI = \rho b\Delta x^2\,dx$$

$$\therefore I = \int_{x=-a/2}^{x=a/2}\rho b\Delta x^2\,dx = \rho b\Delta\left[\frac{x^3}{3}\right]_{-a/2}^{a/2}$$

$$= \frac{\rho b\Delta}{3}\left(\frac{a^3}{8}+\frac{a^3}{8}\right) = \frac{\rho b\Delta a^3}{12} = \frac{Ma^2}{12}$$

(ii)




$I_{edge}$ = MoI about edge $= \frac{Ma^2}{12} + M\left(\frac{a}{2}\right)^2 = \frac{Ma^2}{3}$

(|| axis theorem)

$$M = 1.5^2\times5\times10^{-3}\times7.85\times10^3 = 88.3\,\mathrm{kg} \quad \therefore I = 66.2\ \mathrm{kg\,m^2}$$

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$$3\quad I = \int_{-R}^{R}\frac{1}{2}\rho\pi(R^2-x^2)^2\,dx = \frac{\rho\pi}{2}\int_{-R}^{R}(R^4-2x^2R^2+x^4)\,dx$$

$$= \frac{\rho\pi}{2}\left[R^4x - \frac{2}{3}R^2x^3 + \frac{x^5}{5}\right]_{-R}^{R}$$

$$= \frac{\rho\pi}{2}\left(R^5 - \frac{2}{3}R^5 + \frac{R^5}{5} + R^5 - \frac{2}{3}R^5 + \frac{R^5}{5}\right) = \rho\pi R^5 \frac{8}{15}$$

But $M = \rho \frac{4}{3}\pi R^3 \quad \therefore I = \frac{2}{5}MR^2$

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4 (i) $\omega_{init} = \frac{225 \times 2\pi}{60} = 23.6 \text{ rad s}^{-1}$

$\omega_{final} = \frac{112 \times 2\pi}{60} = 11.7 \text{ rad s}^{-1}$

$|\Delta K| = \frac{1}{2} I (\omega_{init}^2 - \omega_{final}^2)$

$= \frac{1}{2} \times 1.35 \times 10^7 \times 4.18 \times 10^2 = 2.82 \times 10^9 \text{ J}$

$\therefore$ Power $= \frac{|\Delta K|}{\Delta t} = \frac{2.82 \times 10^9}{7} \doteq 4.03 \times 10^8 \text{ W}$

$\therefore$ total power (both flywheels) $= 805 \text{ MW}$

(ii) If the mass was distributed uniformly then:
$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 7.8 \times 10^5 \times (4.5)^2 = 7.90 \times 10^6 \text{ kg m}^2$

If the mass was concentrated into a thin shell at rim then:
$I = MR^2 = 1.58 \times 10^7 \text{ kg m}^2$
The actual value is between these extremes, but closer to the latter. The mass is mainly concentrated near the rim

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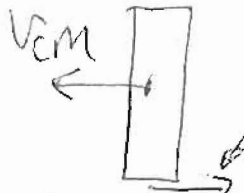
5 (i) About CoM: $\frac{dL}{dt} = \tau = aF \quad$ But $L = I\omega$

$\therefore \frac{dL}{dt} = \underbrace{I}_{ml^2/12} \frac{d\omega}{dt} \quad \therefore \frac{d\omega}{dt} = \frac{12aF}{ml^2}$


At end of rod $v = \frac{\omega l}{2} \quad \therefore \frac{dv}{dt} = \frac{l}{2}\frac{d\omega}{dt} = \frac{6aF}{ml}$

(ii) If CoM can move : $M\frac{dv_{cm}}{dt} = F$




$\frac{\omega l}{2}$ wrt CoM ∴ vel of lower end of rod $= \frac{\omega l}{2} - V_{cm}$

$\therefore \frac{dv}{dt} = \frac{l}{2}\frac{d\omega}{dt} - \frac{dv_{cm}}{dt} = \frac{6aF}{ml} - \frac{F}{m}$

(iii) Minimize shock → want $\frac{dv}{dt} = 0 \Rightarrow \frac{6a}{l} - 1 = 0 \Rightarrow a = l/6$

i.e. sweet spot is 0.178m above centre of bat
→ 35.7 cm from top.

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6 (i)


About end: $\frac{dL}{dt} = \tau = Mg\underbrace{\frac{l}{2}\sin\theta}_{\text{lever arm}}$

But $L = I\omega$

MoI about end $= Ml^2/3$

$\therefore \frac{\cancel{M}l^2}{3}\frac{d\omega}{dt} = \cancel{M}g\frac{l}{2}\sin\theta \quad \therefore \alpha = \frac{3g\sin\theta}{2l}$

(ii) $a = \frac{dv}{dt} = l\frac{d\omega}{dt} \quad \theta = 90° \Rightarrow a = l.\frac{3g\sin 90°}{2l} = \frac{3}{2}g$
$= 14.7 ms^{-1}$

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7 $\omega_p$ = ang speed of person wrt ground
$\omega_r$ = - - - - - roundabout wrt ground.
$v^*$ = speed of person wrt ground $= R\omega_p$
$v$ = speed of person wrt roundabout.
Roundabout moving at speed $R\omega_r$ wrt ground.
$\therefore v^* = v + R\omega_r$
$L_{tot}$ = tot ang mom of person + roundabout $= mR^2\omega_p + \frac{1}{2}MR^2\omega_r$

$L_{tot} = 0$ initially & stays $= 0$ when person is moving
$\therefore \omega_r = -2\frac{m}{M}\omega_p \quad \therefore R\omega_r = -2\mu R\omega_p \quad \therefore v^* = v - 2\mu v^*$

($\frac{m}{M} = \mu$; $R\omega_p = v^*$)

$\therefore v^*(1+2\mu) = v \qquad \therefore v^* = v/(1+2\mu)$

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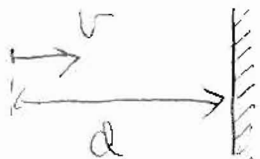
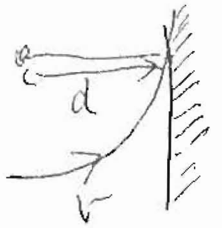
8. 

Suppose Ms Reckless is travelling at vel $v$ towards the wall, distance $d$ from it

Work needed to stop car in straight line:
$W = K_{final} - K_{initial} = -\frac{1}{2}mv^2$ ($m$ = mass of car + Ms R)

Motion in straight line, force in opposite dirn to motion
$\therefore W = -Fd \rightarrow F = \frac{mv^2}{2d}$



To just avoid wall would need to turn in arc of circle of radius $d$. Assuming speed stays const, centripetal force needed to do this is $\frac{mv^2}{d}$

i.e. to turn needs $2\times$ force needed to stop in straight line
$\Rightarrow$ best to carry on straight towards the wall.

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