Mechanics
First Year Physics | M. Coppins
30.11.07

Problem Sheet 7: Lectures 4.2 and 4.3

Exercises

1. A cd is spinning at 400 rpm (revolutions per minute). Use the data below to calculate its kinetic energy and magnitude of angular momentum (about its centre).

   Mass of cd = 15 g.

   Radius of cd = 60 mm.

   Radius of hole in cd = 7.5 mm.

2. (i) Consider a flat rectangular plate of sides a and b, and mass M. Show that the moment of inertia of the plate about an axis through the centre of mass, parallel to the side of length b is

   $$I = \frac{Ma^2}{12} \; .$$

   (ii) Calculate the moment of inertia of a square steel plate (sides of length 1.5 m, thickness 5 mm) about an axis along one side. The density of steel is 7.85×10^3 kg m^{-3}.

3. In Lecture 4.2 we found that the moment of inertia of a solid sphere of radius R and uniform density ρ is given by

   $$I = \int_{-R}^{+R} \frac{1}{2}\rho\pi(R^2 - x^2)^2 dx \; .$$

   Carry out the integral to show that

   $$I = \frac{2}{5}MR^2$$

   where M is the mass of the sphere.

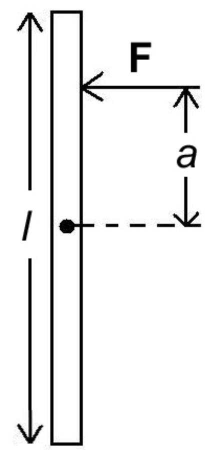
4. (i) The Joint European Torus (JET) at Culham in Oxfordshire is currently the biggest fusion experiment in the world. Because it uses large amounts of energy in short bursts, energy is stored on-site in two large flywheels. Each flywheel has a moment of inertia of 1.35×10^7 kg m^2, and at full speed rotates at 225 rpm. The flywheels can be used to generate electricity. Doing so extracts energy from them and they slow down. Given that they can be slowed to 112 rpm in 7 s, calculate the power delivered by the two flywheels together during that period.

   (ii) Given that each flywheel has a mass of 7.8×10^5 kg and a radius of 4.5 m, what qualitative conclusion can you draw about the distribution of mass within the flywheel?

Problems

5. (i) A uniform rod of length l and mass M is pivoted about its centre. Force $\mathbf{F}$ is applied normal to the rod at a point distance a from the centre (as indicated in the figure). Show that the magnitude of the acceleration of the end of the rod furthest from the point of application of the force is is given by:

$$\frac{dv}{dt} = \frac{6aF}{Ml} \ .$$



(ii) If the rod is not pivoted but free to move, then the force will not only rotate the rod but also accelerate the centre of mass. Show that in this case the magnitude of the acceleration of the end of the rod furthest from the point of application of the force is is given by:

$$\frac{dv}{dt} = \frac{F}{m}\left(\frac{6a}{l} - 1\right) \ .$$

(iii) The "sweet spot" of a bat or racquet is the point where the ball's impact causes the smallest shock to the players hands. Find the location of the sweet spot of a baseball bat, assuming that the bat can be treated as a uniform rod of length 1.07 m, and that it is being held vertically, exactly at the lower end.[1]

6. (i) A uniform rod of length l is pivotted freely at one end, and allowed to fall under gravity. Show that the angular acceleration of the rod when it is at angle θ to the vertical is $\alpha = \dfrac{3g\sin\theta}{2l}$.

(ii) What is the downward tangential acceleration of the free end of the rod (in m s^{-2}) at the instant the rod is horizontal?

7. A person of mass m stands at the edge of a roundabout which consists of a uniform disc of radius R and mass M. She starts to walk round the circumference at a speed v with respect to the roundabout. Assuming that the roundabout can rotate freely without friction, show that her speed with respect to the ground is given by:

$$v^* = \frac{v}{1 + 2\mu}$$

where $\mu = m/M$.

[1] The sweet spot considered here is often called the centre of percussion. There is another sweet spot where the impact does not excite vibrations in the bat.

8. [This question should really have been on Problem Sheet 5.] Ms Reckless (who, as you may remember, enjoys driving fast in reduced visibility) is spending a foggy Christmas day driving her car around the deserted car park of her local supermarket. Suddenly she becomes aware that the wall of the supermarket is straight ahead, aligned perpendicular to her direction of motion. Should she brake and try to stop while continuing to drive in a straight line, or turn to avoid colliding with the wall. [Think about the forces needed to stop or to turn in an arc of a circle.]

Numerical Answers

1. 0.0241 J, 1.15×10^{-3} kg m^2 s^{-1}
2. (ii) 66.2 kg m^2.
4. (i) 805 MW.
5. (iii) 35.7 cm from top.
6. (ii) 14.7 m s^{-2}.