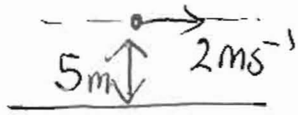
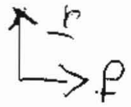
MECHANICS PROBLEM SHEET 6, ANSWERS

1. (i)(a)



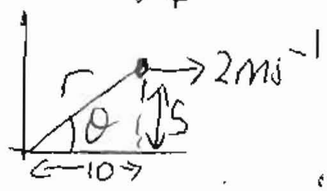
$|\underline{p}| = 3\times2 = 6\ \mathrm{kg\,m\,s^{-1}}$

$L = |\underline{r}\times\underline{p}| = 5\times6\times\sin 90^\circ = 30\ \mathrm{kg\,m^2 s^{-1}}$



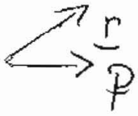
$\underline{r}\times\underline{p} \rightarrow \otimes = -z$ dirn i.e. $\underline{L} = -30\underline{k}\ \mathrm{kg\,m^2 s^{-1}}$

(b)



$L = r\times6\times\sin\theta$ But $r\sin\theta = 5$

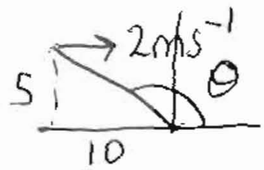
$\therefore L = 30\ \mathrm{kg\,m^2 s^{-1}}$ again



$\underline{r}\times\underline{p} \rightarrow -z$ dirn again

$\Big\}$ same $\underline{L}$

(c)

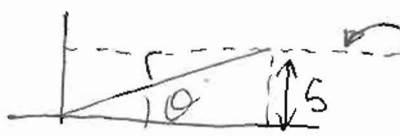


$r\sin\theta = 5 \Rightarrow L = 30\ \mathrm{kg\,m^2 s^{-1}}$ again

$\underline{r}\times\underline{p} \rightarrow -z$ dirn again

$\Big\}$ same $\underline{L}$

(d)



for any pt along this line $r\sin\theta = 5$ & dirn is always $-\underline{k}$ → same $\underline{L}$ for any pt on line.

(ii) There is no torque acting on the particle so its ang mom must be const.

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2. Circular motion: $L = m\omega r^2$

For Earths orbit $\omega = \frac{2\pi}{T}$ period = 1 year

$$\therefore L = \frac{5.98\times10^{24}\times2\times\pi\times(1.49\times10^{11})^2}{365.25\times24\times60\times60} = 2.64\times10^{40}\ \mathrm{kg\,m^2 s^{-1}}$$

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3 (i) $\underline{L} = \underline{r} \times \underline{p} = 10^{-3} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -4 & 0 \\ 5 & 2 & -3 \end{vmatrix} = 10^{-3}(12\underline{i} + 6\underline{j} + 24\underline{k})\ kg\,m^2 s^{-1}$

(ii) (a) $\underline{r} = 4\underline{i} - \underline{j} + 6\underline{k}$

$$\underline{\tau} = \underline{r} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & -1 & 6 \\ 10 & -10 & 10 \end{vmatrix} = 50\underline{i} + 20\underline{j} - 30\underline{k}\ Nm$$

(b) Shift origin to (1,2,3)

New pos vec: $\underline{r} = 4\underline{i} - \underline{j} + 6\underline{k} - (\underline{i} + 2\underline{j} + 3\underline{k}) = 3\underline{i} - 3\underline{j} + 3\underline{k}$

$$\underline{\tau} = \underline{r} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -3 & 3 \\ 10 & -10 & 10 \end{vmatrix} = 0$$

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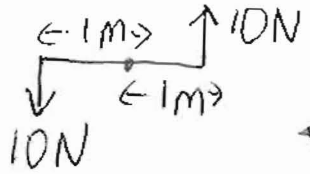
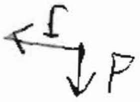
4 (i) (a) $\underline{F}_{net} = -10\underline{j} + 10\underline{j} = 0$

(b) 

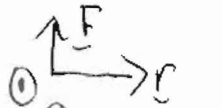
$\underline{\tau}_L$ = torque due to force at LH end



$\odot\ \underline{\tau}_L \leftarrow$ in +z dim.

$|\underline{\tau}_L| = 1 \times 10 = 10\ NM$

$\underline{\tau}_R$ = torque due to force at RH end

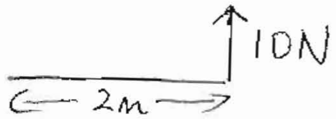


$\underline{\tau}_R \leftarrow$ in +z dim

$|\underline{\tau}_R| = 1 \times 10 = 10\ NM$

$\Big\}$ total torque $= 20\underline{k}\ NM$

(c) Now $\underline{\tau}_L = 0$ (lever arm = 0)



τ_R in +z dim

$|\underline{\tau}_R| = 2 \times 10 = 20N \Rightarrow$ total torque $= 20\underline{k}\ NM$

(d) $\underline{\tau}_R = 0$

$\rightarrow$ total torque $= 20\,\underline{k}$ Nm

(ii) No. The torque about a given pt is zero if the lever arm is zero i.e. the line of action of the force passes thro' the given pt. In this case the lines of action of the forces are parallel so do not pass thro' the same pt.

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5 (i) Circular orbit $\rightarrow$ centripetal acc $= -\dfrac{v^2}{r}\,\hat{\underline{r}}$

$$\underline{F} = m\underline{a} \rightarrow -\frac{e^2}{4\pi\epsilon_0 r^2}\,\hat{\underline{r}} = -\frac{m_e v^2}{r}\,\hat{\underline{r}} \quad \therefore v = \frac{e}{\sqrt{4\pi\epsilon_0 m_e r}}$$

(ii) $$L_n = m_e v r_n = \frac{m_e r_n e}{\sqrt{4\pi\epsilon_0 m_e r_n}} = e\sqrt{\frac{m_e r_n}{4\pi\epsilon_0}}$$

But $L_n = n\hbar \rightarrow e\sqrt{\dfrac{m_e r_n}{4\pi\epsilon_0}} = n\hbar \rightarrow r_n = \dfrac{n^2\hbar^2 4\pi\epsilon_0}{m_e e^2}$

(iii) diameter $= 2r_1 = \dfrac{2\times 1^2 \times (1.05\times10^{-34})^2 \times 4\times\pi\times 8.85\times10^{-12}}{9.11\times10^{-31}\times(1.60\times10^{-19})^2}$

$= 1.05\times10^{-10}$ m

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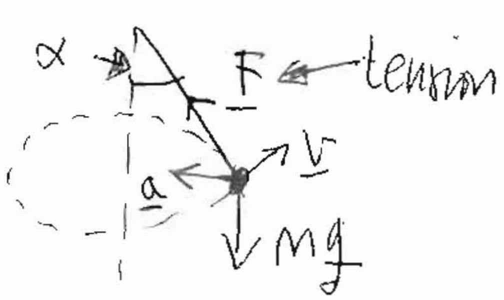
6 Bob moves in circle of radius $r = l\sin\alpha$



Resolve forces

- No vertical accel $\rightarrow F\cos\alpha - mg = 0 \rightarrow F = \dfrac{mg}{\cos\alpha}$
- Horizontal accel $\underline{a}$ towards centre of circle

$|\underline{a}| = \omega^2 r \quad \therefore \quad F\sin\alpha = m\omega^2 r$

Subs for F & r $\rightarrow \dfrac{mg}{\cos\alpha}\sin\alpha = m\omega^2 l \sin\alpha$

$\rightarrow \omega^2 = \dfrac{g}{l\cos\alpha} \quad \therefore \ T = \text{period} = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{l\cos\alpha}{g}}$

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7 (i) $\underline{a} = \dfrac{d\underline{v}}{dt} = \dfrac{dv_r}{dt}\hat{\underline{r}} + v_r \underbrace{\dfrac{d\hat{\underline{r}}}{dt}}_{\omega\hat{\underline{\theta}}} + \underbrace{\dfrac{dr}{dt}}_{v_r}\omega\hat{\underline{\theta}} + r\underbrace{\dfrac{d\omega}{dt}}_{\alpha}\hat{\underline{\theta}} + r\omega\underbrace{\dfrac{d\hat{\underline{\theta}}}{dt}}_{-\omega\hat{\underline{r}}}$

$= \left(\dfrac{dv_r}{dt} - \omega^2 r\right)\hat{\underline{r}} + (2\omega v_r + r\alpha)\hat{\underline{\theta}}$

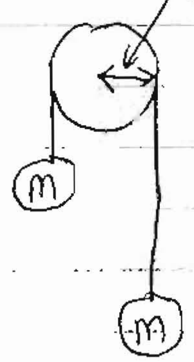
(ii) $F_\theta = m(2\omega v_r + r\alpha) \quad \therefore \ \tau = mr(2mv_r + r\alpha)$

(iii) $L = mrv_\theta = mr^2\omega$

$\dfrac{dL}{dt} = m2r\underbrace{\dfrac{dr}{dt}}_{v_r}\omega + mr^2\underbrace{\dfrac{d\omega}{dt}}_{\alpha} = mr(2\omega v_r + r\alpha) = \tau$

8

d = radius of pulley

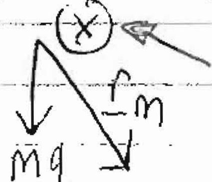


Torque exerted by weight of bananas about centre of pulley $= d\,mg$

lever arm

$\underline{\tau}_b = \underline{r}_b \times (m\underline{g}) \rightarrow$ out of paper

Torque exerted by weight of monkey about centre of pulley $= d\,mg$



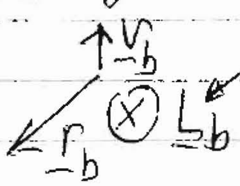
$\underline{\tau}_m = \underline{r}_m \times (m\underline{g}) \rightarrow$ into paper

$$\sum \underline{\tau}^{ext} = \underline{\tau}_b + \underline{\tau}_m = 0 \quad \text{(equal \& opposite)}$$

$\therefore$ $\underline{L}_{tot}$ = const (about centre of pulley)
Initially at rest: $\underline{L}_{tot} = 0$

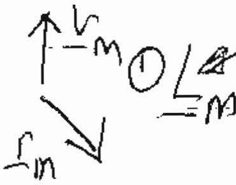
Define $\underline{v}_b$ = ban's vertical vel, $\underline{v}_m$ = monkey's vertical vel.
About centre of pulley

$L_b = m d v_b$



into paper

$L_m = m d v_m$



out of paper

But $\underline{L}_{tot} = \underline{L}_b + \underline{L}_m = 0$

$\therefore$ $\underline{v}_b = \underline{v}_m$ at all times. i.e. if monkey goes up/down the ban's also go up/down at same vel.

Monkey climbs rope i.e. length of rope between monkey & ban's decreases $\therefore$ both monkey & ban's go up, & vertical separation between them stays const.