Mechanics
First Year Physics

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Problem Sheet 6: Lectures 3.6–4.1

Exercises

1. (i) A particle of mass 3 kg moves with a constant velocity of +2 m s^{-1} in the x direction along the line $y = 5$. Find its angular momentum (vector) about the origin when:
   (a) $x = 0$
   (b) $x = 10$ m
   (c) $x = -10$ m
   (d) $x = 10^{25}$ m

   (ii) You should have found that the angular momentum was the same in all four cases in part (i). Could you have predicted this before doing any calculations?

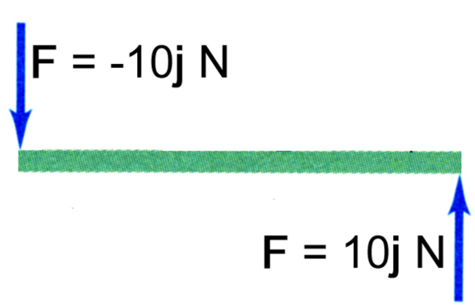
2. Calculate the magnitude of the angular momentum of the Earth about the Sun. Ignore the rotation of the Earth about its axis and assume a circular orbit of radius 1.49×10^{11} m. The mass of the Earth is 5.98×10^{24} kg.

3. (i) A particle of mass 10^{-3} kg has a velocity $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ m s^{-1}. Calculate its angular momentum about the origin when its position vector is $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j}$.

   (ii) A force $\mathbf{F} = 10\mathbf{i} - 10\mathbf{j} + 10\mathbf{k}$ N is applied at the point $(4, -1, 6)$. Calculate the torque about
   (a) the origin,
   (b) the point (1,2,3).

4. (i) The figure below shows a rod of length 2 m which is aligned along the x axis. Forces are applied at the ends of the rod as shown. Find
   (a) the net force on the rod,
   (b) the torque about the centre of the rod,
   (c) the torque about the left-hand end of the rod, and
   (d) the torque about the right-hand end of the rod.

   (ii) Is there any point about which the torque is zero?



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Problems

5. The Bohr model of the atom assumes that the atomic electrons orbit the nucleus in stable, circular orbits. In this model a Hydrogen atom consists of a single electron in orbit around a proton

   (i) The force on an electron distance r from a proton is given by $\mathbf{F} = -\frac{e^2}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$ where $\hat{\mathbf{r}}$ is the unit vector pointing from the proton to the electron. Show that the speed of an electron in an orbit of radius r about a proton is given by

   $$v = \frac{e}{\sqrt{4\pi\epsilon_0 m_e r}}$$

   where m_e is the electron mass.

   (ii) The Bohr model also assumes that L, the magnitude of the electrons orbital angular momentum about the nucleus is quantized: the only orbits which are allowed are those for which $L = n\hbar$, where n is an integer (called the principal quantum number) and $\hbar$ is a constant. Show that the radius of the n^{th} orbit in a Hydrogen atom is given by

   $$r_n = n^2 \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2} \ .$$

   (iii) Under normal conditions the electron will be in the *ground state* (the $n = 1$ orbit). Use the data below to calculate the diameter of a Hydrogen atom in its ground state:
   $\hbar = 1.05 \times 10^{-34}$ kg m^2 s^{-1}
   $\epsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}
   $m_e = 9.11 \times 10^{-31}$ kg
   $e = 1.60 \times 10^{-19}$ C.

6. Like the simple pendulum, the *conical pendulum* consists of a bob of mass m suspended by a massless string of length l. Instead of swinging backwards and forwards, however, the bob moves in a horizontal circle with a constant speed v, the string making an angle α with the vertical direction. As the bob moves the string traces out a cone; hence the name. Find expressions for the tension in the string and the period, in terms of m, l, α and g. (Probably the hardest thing about this question is trying to think of different symbols for tension and period.)

7. (i) In lecture 3.5 we showed that the velocity of a particle moving in a plane can be written in polar coordinates as $\mathbf{v} = v_r\hat{\mathbf{r}} + r\omega\hat{\theta}$ where $v_r = \dfrac{dr}{dt}$ and $\omega = \dfrac{d\theta}{dt}$. By differentiating this expression with respect to time show that the acceleration of the particle is

$$\mathbf{a} = \left(\frac{dv_r}{dt} - \omega^2 r\right)\hat{\mathbf{r}} + (2\omega v_r + r\alpha)\,\hat{\theta}$$

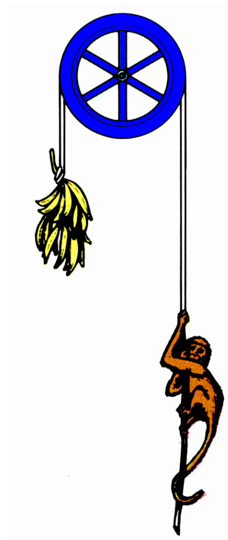
where $\alpha = \dfrac{d\omega}{dt}$. [This is the generalization of the expression found in section 3.5.4, which applied only to circular motion.]

(ii) The magnitude of the torque acting on the particle can be written $\tau = rF_\theta$. show that $\tau = mr\,(2\omega v_r + r\alpha)$, where m is the mass of the particle.

(iii) The magnitude of the angular momentum of the particle can be written $L = mrv_\theta$. Show that $\dfrac{dL}{dt} = \tau$.

8. *The following problem was originally posed by Lewis Carroll. He couldn't solve it.*

A monkey is hanging onto a massless rope which passes over a frictionless, massless pulley. At the other end of the rope is a bunch of bananas, with a mass exactly equal to that of the monkey. Thus the monkey and the bananas are in equilibrium. The monkey is initially lower (see figure) and he starts to climb the rope to reach the bananas. Does the bunch of bananas go up, go down, or stay where it is? Does the monkey go up, go down, or stay where he is? [Hint: think about: (i) the total angular momentum of the monkey and bananas, (ii) the total torque exerted by external forces.]



Numerical Answers

1. (i) $-30\mathbf{k}$ kg m^2 s^{-1} for all four cases

2. 2.64×10^{40} kg m^2 s^{-1}

3. (i) $(12\mathbf{i} + 6\mathbf{j} + 24\mathbf{k}) \times 10^{-3}$ kg m^2 s^{-1}

   (ii) (a) $50\mathbf{i} + 20\mathbf{j} - 30\mathbf{k}$ N m, (b) 0

4. (i) (a) 0, (b)–(d) $20\mathbf{k}$ N m

5. (iii) 1.05×10^{-10} m