Mechanics
First Year Physics

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Problem Sheet 5: Lectures 3.3–3.5

1. (i) Figure 1 shows a letter L made up of 6 squares, each of side 0.1 m. The lower left-hand corner of the L is at the origin. Find the coordinates of its centre of mass.

   (ii) Figure 2 shows a flat disc of radius 0.5 m, whose centre is at the origin. A hole of radius 0.25 m has been cut in it. Find the coordinates of the centre of mass.

   (iii) The mass of the Earth is 5.98×10^{24} kg. The mass of the Sun is 1.99×10^{30} kg. The mean Earth-Sun distance is 1.49×10^{11} m. How far is the centre of mass of the Earth-Sun system from the centre of the Sun?

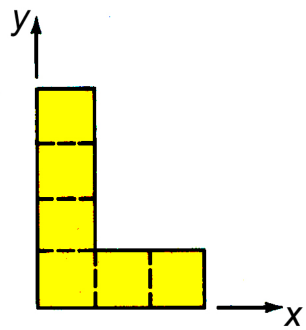


Fig 1

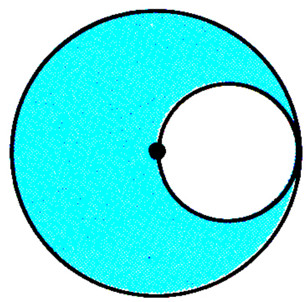


Fig 2

2. The figure below shows a simple toy. When the double cone is placed on the incline it appears to roll up it. Why does it do that?

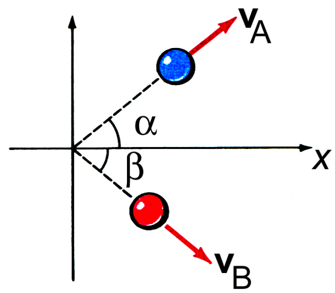
3. (i) A particle is executing uniform circular motion with a radius of 0.2 m and a period of 5 s. Calculate its angular speed, speed, and the magnitude of its acceleration.

   (ii) The particle's angular velocity is in the $+z$ direction. At some instant its acceleration is in the $-x$ direction. What direction is the particle moving in at that instant?

4. Show that the rate at which work is done (= power = $P = \frac{dW}{dt}$) is zero for an object whose acceleration is perpendicular to its velocity.

Continued overleaf

5. In a game of billiards the cue ball (A), moving with speed u_A in the $+x$ direction, hits the stationary red ball (B). Immediately after the collision the cue ball is moving with speed v_A at angle of α and the red ball is moving with a speed v_B at an angle $-\beta$ (both angles with respect to the x axis; see diagram).



(i) Assuming that the balls have the same mass, and that the collision is elastic, show that

$$\begin{aligned} v_A \cos\alpha + v_B \cos\beta &= u_A \\ v_A \sin\alpha - v_B \sin\beta &= 0 \\ v_A^2 + v_B^2 &= u_A^2 \end{aligned}$$

(ii) In one such collision the speed of the balls immediately after the collision are $v_A = 2.0\ \mathrm{m\,s^{-1}}$ and $v_B = 1.0\ \mathrm{m\,s^{-1}}$. Show that $\sin\beta = 2.0\sin\alpha$, and, $\cos\beta = \sqrt{5} - 2.0\cos\alpha$. Hence, or otherwise, calculate α and β.

(iii) For the situation considered in part (ii) you should have found that $\alpha + \beta = 90^o$. This is true in general for an elastic collision between two billiard balls, one of which is initially at rest, and is known to aficionados of games involving poking balls with wooden sticks (billiards, snooker, pool, etc) as the "90^o rule". Prove it.

(iv) In 1869 John W. Hyatt invented celluloid, the world's first plastic. Amongst other things, he used it to make billiard balls, which had previously been made from ivory. Unfortunately, celluloid is highly inflammable, as was the paint used on billiard balls, and games of billiards in the late nineteenth century were frequently enlivened by the balls exploding. In one memorable, wild-west incident, an exploding billiard ball precipitated a gunfight in a Colorado gaming saloon. Consider an event of of this kind in which the cue ball explodes into two equal fragments on impact with the initially stationary red ball. The latter remains intact after the collision. Assume that the two fragments of the cue ball and the red ball stay on the table. Immediately after the explosion both the cue ball fragments have a speed of 10 m $\mathrm{s^{-1}}$, one moving at 10.0^o, the other at 180^o, to the original direction of motion. The red ball acquires a speed of 3.0 m $\mathrm{s^{-1}}$. Given that the cue ball and the red ball each have a mass of 0.17 kg, calculate the energy released in the explosion.

6. A woman of mass 50 kg stands at one end of a plank of length 20 m. The mass of the plank is 20 kg. It is at rest on a frozen pond, and the friction between the plank and the ice is negligible. The woman walks to other end of the plank. How far does she move relative to the pond? [Hint: there is no net external force acting on the plank/woman system.]

7. Consider two objects with masses m_A and m_B moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$.

   (i) Show that $V^2 = v_A^2 - 2\mathbf{v}_A.\mathbf{v}_B + v_B^2$ where $\mathbf{V} = \mathbf{v}_A - \mathbf{v}_B$.

   (ii) Show that $v_{CM}^2 = \frac{1}{M^2}\left(m_A^2 v_A^2 + 2m_A m_B \mathbf{v}_A.\mathbf{v}_B + m_B^2 v_B^2\right)$ where $\mathbf{v}_{CM}$ is the centre of mass velocity and $M = m_A + m_B$.

   (iii) In Classwork IV we found that the total kinetic energy in the centre of mass frame can be written $K'_{tot} = \frac{1}{2}\mu V^2$ where $\mu = \frac{m_A m_B}{M}$. Show that $K'_{tot} + K_{cm} = K_{tot}$, where $K_{cm} = \frac{1}{2}M v_{CM}^2$ and K_{tot} is the total kinetic energy in the "lab frame". [This equation tells us that the total kinetic energy of two particles can be expressed as the kinetic energy of the centre of mass + the kinetic energy of the particles about the centre of mass.]

8. You can twirl a bucket of water in a vertical circle without the water spilling out.

   (i) What are the forces acting on the water at the highest point?

   (ii) Why doesn't it spill out?

Numerical Answers

1. (i) (0.1,0.15)

   (ii) (-0.083,0)

   (iii) 448 km.

3. 1.26 rad^{-1}, 0.25 m s^{-1}, 0.32 m s^{-2}.

5. (ii) $\alpha = 26.6^o$, $\beta = 63.4^o$

   (iv) 8.60 J.

6. 5.71 m