MECHANICS PROBLEM SHEET 4 : ANSWERS

1 (i) (a) true, (b) false, (c) true, (d) false

(ii) (a) $\underline{A} = -\underline{B}$ (b) $\underline{A}$ & $\underline{B}$ in same dirn
(c) $\underline{B}$ in opposite dirn to $\underline{A}$ & $A > B$
(d) $\underline{B}$ in opposite dirn to $\underline{A}$ & $B > A$
(e) $\underline{A} \perp$ to $\underline{B}$

(iii) $(\underline{i}+\underline{j}+\underline{k})/\sqrt{3}$

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2 (i) $|\underline{v}_A| = \sqrt{2^2+2^2} = \sqrt{8}\ ms^{-1}$
(ii) $|\underline{v}_B| = \sqrt{1+9} = \sqrt{10}\ ms^{-1}$
(iii) $\underline{v}_B - \underline{v}_A = -3\underline{i} + 2\underline{j} + 3\underline{k}\ ms^{-1}$
(iv) $\underline{v}_A - \underline{v}_B = 3\underline{i} - 2\underline{j} - 3\underline{k}\ ms^{-1}$

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3 (i) $\underline{v} = \frac{d\underline{r}}{dt} = 10t\underline{i} - 2\underline{j} + 3t^2\underline{k}$, $\underline{a} = \frac{d\underline{v}}{dt} = 10\underline{i} + 6t\underline{k}$

(ii) Direction of motion at any instant is direction of $\underline{v}$ at that instant
$\underline{v}(t=2) = 20\underline{i} - 2\underline{j} + 12\underline{k}$
$|\underline{v}(t=2)| = \sqrt{20^2+2^2+12^2} = 2\sqrt{137}$
$\therefore$ unit vec in dirn of $\underline{v}(t=2) = \dfrac{10\underline{i} - \underline{j} + 6\underline{k}}{\sqrt{137}}$

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4 (i) Horizontal motion at const vel $\Rightarrow v_x = v_{0x} = 250\ ms^{-1}$
Vertical motion at const accel, $v_{0z} = 0 \Rightarrow v_z = -gt$

$\therefore z = 300 - \frac{1}{2}gt^2 = 0$ when $t^2 = 600/g \Rightarrow t = 7.82s$

Horiz. distance travelled (to target) $= v_x t = 1.96 \times 10^3 m$

(ii) At impact $v_x = 250\ ms^{-1}$, $v_z = -9.81 \times 7.82 = -76.7\ ms^{-1}$

$\therefore \tan\theta = \frac{76.7}{250} \Rightarrow \theta = 17.1^\circ$

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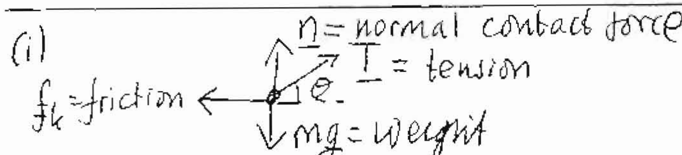
5 (i)



Const vel: $\underline{F} = 0 \Rightarrow$ vertical: $n + T\sin\theta - mg = 0$ (1)
horiz.: $T\cos\theta - f_k = 0$ (2)
But $f_k = \mu_k n$ $\therefore$ (2) $\Rightarrow n = \frac{T\cos\theta}{\mu_k}$
Subs into (1) $\rightarrow \frac{T\cos\theta}{\mu_k} + T\sin\theta = mg \rightarrow T = \dfrac{\mu_k mg}{(\cos\theta + \mu_k \sin\theta)}$

(ii) As θ is increased from zero the vertical comp of the tension increases. This is upwards & $\therefore$ reduces n. But $f_k \propto n$ $\therefore$ f_k falls, allowing smaller T.

(iii) T is minimum when $\cos\theta + \mu_k \sin\theta$ is maximum
$\frac{d}{d\theta}(\cos\theta + \mu_k \sin\theta) = -\sin\theta + \mu_k\cos\theta$
$= 0$ when $\sin\theta = \mu_k\cos\theta$
$\Rightarrow \tan\theta = \mu_k$

(iv) (a) $\tan^{-1}(0.4) = 21.8^\circ$ $\therefore T_{min} = \dfrac{0.4 \times 10 \times 9.81}{\cos(21.8^\circ) + 0.4\sin(21.8^\circ)} = 36.4N$

(b) For $\theta = 0$: $T = \mu_k mg = 39.2N$

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6 $\underline{F} = \alpha\underline{i} + \beta t^2\underline{j}$ $\therefore \underline{a} = \frac{\alpha}{m}\underline{i} + \frac{\beta t^2}{m}\underline{j}$

Integrate: $\underline{v} = \underline{v}_0 + \frac{\alpha t}{m}\underline{i} + \frac{\beta t}{3m}\underline{j}$ $\quad \underline{v}_0 = \text{int. const} = 0$ (initially at rest)

Integrate again: $\underline{r} = \underline{r}_0 + \frac{\alpha}{2m}t^2\underline{i} + \frac{\beta t^4}{12m}\underline{j}$

$\underline{r}_0$ = int const = 0 (initially at origin)

$\therefore \underline{r} = \frac{t^2}{2m}\left(\alpha\underline{i} + \frac{\beta t^2}{6}\underline{j}\right)$

$\therefore$ distance from origin $= |\underline{r}| = \frac{t^2}{2m}\left(\alpha^2 + \frac{\beta^2 t^4}{36}\right)^{\frac{1}{2}}$

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7 $a_y = \frac{1.60\times10^{-19}\times 500}{1.67\times10^{-27}} = 4.79\times10^{10}\,ms^{-2}$

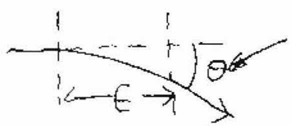
Define $t = 0$ as time proton enters E field

v_x not affected

$\therefore t$ = time proton leaves E field $= \frac{\text{width of region}}{v_x} = \frac{0.5}{10^5} = 5.0\times10^{-6}\,s$

$\therefore v_y(t) = a_y t = 4.79\times10^{10}\times5.0\times10^{-6} = 2.40\times10^5\,ms^{-1}$

$\therefore \underline{v}(t) = 10^5\underline{i} + 2.40\times10^5\underline{j}\ ms^{-1}$



$\tan\theta = \frac{v_y}{v_x} = 2.40 \Rightarrow \theta = 67.3^\circ$

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8 (i) (a) Define $t = 0$ as instant lift starts to accelerate & $z = 0$ as position of floor of lift at $t = 0$

z_b = height of bolt at time $t = z_0 - \frac{1}{2}gt^2$ $\quad$ ($z_0 = 3m$, $g = 9.81\,ms^{-2}$)

z_f = height of floor of lift at time $t = \frac{1}{2}at^2$ $\quad$ ($a = 2\,ms^{-2}$)

Bolt hits floor when $z_b = z_f$

$\therefore 3 - \frac{1}{2}\times9.81t^2 = \frac{1}{2}\times2\times t^2 \rightarrow 5.90t^2 = 3 \Rightarrow t = 0.71s$

(b) $|\underline{v}_b| = gt = 9.81\times0.71 = 6.99\,ms^{-1}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}\times0.5\times(6.99)^2 = 12.2J$

(d) $\underline{v}_{lift} = at\,\underline{k} = 2\times0.71\underline{k} = 1.62\underline{k}\ ms^{-1}$

(e) $\underline{v}_{bolt} - \underline{v}_{lift} = -6.99\underline{k} - 1.62\underline{k} = -8.62\underline{k}\ ms^{-1}$

(ii) Observer in lift sees bolt moving at $8.62\,ms^{-1}$ (relative to lift) $\therefore K = \frac{1}{2}\times0.5\times(8.62)^2 = 17.7J$

(iii) Pseudo force has magnitude = m×accel, in opposite dirn to accel.

$\therefore$ pseudo force on bolt $= 0.5\times2.0 = 1.0N$ down

(iv) Total force on bolt = weight + pseudo-force (both down) $= 0.5\times9.81 + 1.0 = 5.90N$

$\therefore$ work done $= 5.90\times3.0 = 17.7J = \Delta K$ for lift obs.
