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Problem Sheet 4: Lectures 3.1 and 3.2

Exercises

1. These are a few basic questions about vectors

   (i) True or false:

   (a) the components of a vector depend on the choice of coordinate axes.
   (b) the magnitude of a vector depends on the choice of coordinate axes.
   (c) the magnitude of a vector cannot be less than the magnitude of its largest component.
   (d) if $\mathbf{A}.\mathbf{A} = \mathbf{B}.\mathbf{B}$ then $\mathbf{A} = \pm\mathbf{B}$.

   (ii) Under what circumstances would $|\mathbf{A} + \mathbf{B}|$ equal

   (a) zero,
   (b) $A + B$,
   (c) $A - B$,
   (d) $B - A$,
   (e) $(A^2 + B^2)^{1/2}$.

   (iii) What is the unit vector in the direction of $\mathbf{i} + \mathbf{j} + \mathbf{k}$?

2. Object A is moving with a steady velocity $\mathbf{v}_A = 2\mathbf{i} - 2\mathbf{j}$ m s^{-1}. Object B is moving with a steady velocity $\mathbf{v}_B = -\mathbf{i} + 3\mathbf{k}$ m s^{-1}. Find

   (i) the speed of A,
   (ii) the speed of B,
   (iii) the velocity of B relative to A,
   (iv) the velocity of A relative to B.

3. A particle follows a trajectory in which its position vector as a function of time is $\mathbf{r}(t) = 5t^2\mathbf{i} - 2t\mathbf{j} + t^3\mathbf{k}$.

   (i) Find the velocity and acceleration vectors as functions of time.
   (ii) Calculate the unit vector in the direction of motion at $t = 2$ s.

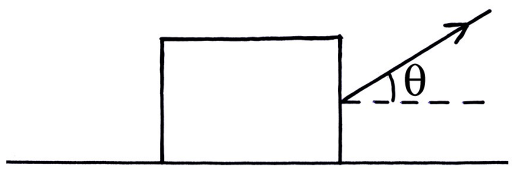
4. A plane is flying horizontally at 250 m s^{-1} at an altitude of 300 m above the ground. The pilot must drop a load of supplies (initially at rest in the plane) so that it hits the ground at a specified target. Assuming air resistance can be neglected, calculate:

   (i) the horizontal distance from the target at which the load must be released, and,
   (ii) the angle to the horizontal at which the load hits the ground.

Continued overleaf

Problems

5. An object of mass m is being pulled across a horizontal floor using a rope which is inclined at an angle θ to the horizontal. The coefficient of kinetic friction between the object and the floor is μ_k. This question concerns the tension in the rope needed to move the object with a *constant velocity.*



   (i) Draw a free body diagram showing the various forces acting on the object, and show that the required tension at angle θ is given by

   $$T = \frac{\mu_k mg}{(\cos\theta + \mu_k \sin\theta)} \ .$$

   (ii) It might be expected that the minimum value of the required tension would occur at $\theta = 0$, for which the horizontal component of the tension is maximum. In fact, as θ is increased from zero, the required value of the tension initially falls. Briefly and qualitatively explain why this happens.

   (iii) Show that the angle at which the required tension is smallest is given by $\tan\theta = \mu_k$.

   (iv) A 10 kg object is being pulled across a horizontal surface at a constant speed. The coefficient of kinetic friction between the object and the surface is 0.4. Calculate

      (a) the minimum tension needed, and

      (b) the tension needed if the rope was horizontal.

6. An object with mass m, initially at rest at the origin, is acted on by a force $\mathbf{F} = \alpha\mathbf{i} + \beta t^2\mathbf{j}$, where α and β are constants. Show that at time t its distance from the origin is given by $|\mathbf{r}| = \dfrac{t^2}{2m}\left(\alpha^2 + \dfrac{\beta^2 t^4}{36}\right)^{1/2}$.

7. The force on an object of charge q in an electric field $\mathbf{E}$ is given by $\mathbf{F} = q\mathbf{E}$. A proton initially moving at $\mathbf{v} = 10^5\mathbf{i}$ m s^{-1} passes through a region in which there is a uniform electric field $\mathbf{E} = 500\mathbf{j}$ V m^{-1}. The width of this region (in the x direction) is 0.5 m. Find the velocity of the proton when it emerges from the region. By what angle is it deflected by the electric field?

   [Proton charge = 1.60×10^{-19} C.

   Proton mass = 1.67×10^{-27} kg.]

8. In Problem Sheet 1 (Q. 8) we considered the motion of a bolt falling from the roof of a lift moving upwards at constant velocity. Now suppose that the lift has a constant upward acceleration of 2 m s^{-2}. Initially it is at rest. The bolt has a mass of 0.5 kg and it starts to fall at the instant that the lift starts to accelerate.

   (i) From the point of view of a stationary observer outside the lift, calculate:
      (a) the time taken for the bolt to reach the floor of the lift,
      (b) the speed of the bolt just before it hits the floor,
      (c) the kinetic energy of the bolt just before it hits the floor,
      (d) the velocity of the lift at the instant the bolt hits the floor,
      (e) the velocity of the bolt relative to the lift just before it hits the floor.

   (ii) From the point of view of an observer in the lift, what is the kinetic energy of the bolt just before it hits the floor?

   (iii) The observer in the lift experiences an extra pseudo-force due to the acceleration. What is the magnitude and direction of the pseudo-force on the bolt?

   (iv) For the observer in the lift the bolt falls a distance of 3 m (from the ceiling to the floor). Show that the work-energy theorem is satisfied for such an observer if the pseudo-force is included when calculating the work done.

Numerical Answers

1. (iii) $(\mathbf{i}+\mathbf{j}+\mathbf{k})/\sqrt{3}$
2. (i) $\sqrt{8}$ m s^{-1}, (ii) $\sqrt{10}$ m s^{-1}, (iii) $-3\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ m s^{-1} (iv) $3\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ m s^{-1}
3. (i) $\mathbf{v} = 10t\mathbf{i} - 2\mathbf{j} + 3t^2\mathbf{k}$ m s^{-1}, $\mathbf{a} = 10\mathbf{i} + 6t\mathbf{k}$ m s^{-2}, (ii) $(10\mathbf{i} - \mathbf{j} + 6\mathbf{k})/\sqrt{137}$
4. (i) 1.96×10^3 m, (ii) 17.1^o
5. (iv) (a) 36.4 N, (b) 39.2 N
7. $\mathbf{v} = 10^5\mathbf{i} + 2.40 \times 10^5\mathbf{j}$ m s^{-1}, angle $= 67.3^o$
8. (i) (a) 0.71 s, (b) 6.99 m s^{-1}, (c) 12.2 J, (d) 1.42 m s^{-1} upwards, (e) 8.42 m s^{-1} downwards, (ii) 17.7 J, (iii) 1.0 N downwards