MECHANICS PROBLEM SHEET 3 ANSWERS

1. $m_A = 0.2\,kg,\ u_A = 5.0\,ms^{-1},\ m_B = 0.3\,kg,\ u_B = 2.0\,ms^{-1}$

mom cons: $0.2v_A + 0.3v_A = 0.2\times5 + 0.3\times2 = 1.6$

Rel. vel changes sign: $v_B - v_A = -(u_B - u_A) = 3 \therefore v_B = v_A + 3$
(Eq 2.5.2.2)

Subs into mom cons: $0.2v_A + 0.3(v_A+3) = 1.6 \rightarrow v_A = \frac{0.7}{0.5} = 1.4\,ms^{-1}$

& $v_B = v_A + 3 = 4.4\,ms^{-1}$

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2 (i) $dp = m\,dv + v_e\,dm \quad \therefore \frac{dp}{dt} = m\frac{dv}{dt} + v_e\frac{dm}{dt}$

But $\frac{dp}{dt} = -F$ ← force opposes motion $\therefore m\frac{dv}{dt} = -v_e\frac{dm}{dt} - F$ ← F_T

(ii) $F_T = -v_e\frac{dm}{dt} = +v_e\left|\frac{dm}{dt}\right| \quad \left(v_e > 0\ \&\ \frac{dm}{dt} < 0\right)$

$F = mg,\ F_T > F \Rightarrow \left|\frac{dm}{dt}\right| > \frac{mg}{v_e}$

$\therefore \left|\frac{dm}{dt}\right|_{min} = \frac{3.06\times10^6\times9.81}{2.65\times10^3} = 1.13\times10^4\,kg\,s^{-1}$

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3 (i) friction $= f = \mu_k n$ ← n = contact force = mg

$= 0.3\times0.5\times9.81$

$= 1.47\,N \quad \Rightarrow F_{net} = 0.53\,N$

$\therefore a = \frac{F_{net}}{m} = \frac{0.53}{0.5} = 1.06\,ms^{-1} \therefore x = \frac{1}{2}at^2 = 0.53\,m$ after 1s

(ii) $W_{person} = 2\times0.53 = 1.06\,J$

(iii) $W_{friction} = -1.47\times0.53 = -0.78\,J$
← opposes motion

(iv) K = total work done $= 1.06 - 0.78 = 0.28\,J$

(v) & (vi) $\bar{P} = \frac{W}{\Delta t} = 1.06\,W$ for person, $-0.78\,W$ for friction

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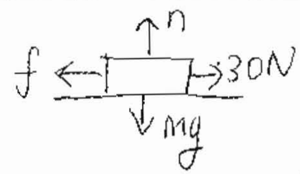
4 (i)



Vertical forces:
weight $= mg = 5\times9.81 = 49.0\,N$ down
n = normal contact force $= 49.0\,N$ up

Horizontal forces: $f_s^{max} = |\mu_s mg| = 34.3\,N$
block is at rest → applied force = 30N in +x dirn
frictional force = 30N in −x dirn

(ii) $f = \mu_k mg = 24.5\,N$ in −x dirn $\therefore a = \frac{30-24.5}{5} = 1.09\,ms^{-1}$

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5 In 1-D elastic coll (see 2.5): $m_A(u_A - v_A) = m_B(v_B - u_B)$ 2.5.2.1

$v_B - v_A = -(u_B - u_A)$ 2.5.2.2

$2.5.2.2 \Rightarrow v_B = u_A - u_B + v_A$ Subs into 2.5.2.1

$m_Au_A - m_Av_A = m_B(u_A - u_B + v_A) - m_Bu_B$

$= m_Bu_A - 2m_Bu_B + m_Bv_A$

$\therefore v_A(m_A + m_B) = m_Au_A - m_Bu_A + 2m_Bu_B \rightarrow v_A = \frac{(m_A - m_B)u_A + 2m_Bu_B}{m_A + m_B}$

Subs back into 2.5.2.2

$v_B = u_A - u_B + \frac{(m_A - m_B)u_A + 2m_Bu_B}{m_A + m_B} = \frac{(m_B - m_A)u_B + 2m_Au_A}{m_A + m_B}$

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6. Mom cons $m_A u_A + m_B u_B = (m_A + m_B)v$ ← after coll both masses move at v

$\therefore \Delta K = \frac{1}{2}(m_A + m_B)v^2 - \frac{1}{2}m_A u_A^2 - \frac{1}{2}m_B^2 u_B^2$

$= \frac{1}{2}\left\{ (m_A+m_B)\frac{(m_A u_A + m_B u_B)^2}{(m_A+m_B)^2} - (m_A u_A^2 + m_B u_B^2) \right\}$ ← from mom cons.

$= \frac{1}{2(m_A+m_B)}\left\{ (m_A u_A + m_B u_B)^2 - (m_A+m_B)(m_A u_A^2 + m_B u_B^2) \right\}$

$= \frac{1}{2(m_A+m_B)}\left\{ m_A^2 u_A^2 + 2m_A m_B u_A u_B + m_B^2 u_B^2 - m_A^2 u_A^2 - m_A m_B u_B^2 - m_B m_A u_A^2 - m_B^2 u_B^2 \right\}$

$= \frac{m_A m_B}{2(m_A+m_B)}\left\{ 2u_A u_B - u_B^2 - u_A^2 \right\} = -$

$= -\frac{1}{2}\mu U^2$ ← $\mu = \frac{m_A m_B}{m_A + m_B}$, $U^2 = (u_A - u_B)^2 = u_A^2 + u_B^2 - 2u_A u_B$

The lost KE goes into the internal energy of the composite body

7 3 coll's take place in rapid succession

(1) 1 with 2 Just before: $u_1 = +1\ ms^{-1}$, $u_2 = 0$
Just after: $v_1 = 0$, $v_2 = +1\ ms^{-1}$

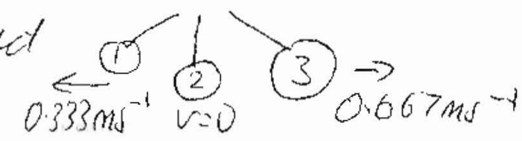
(2) 2 with 3 Just before: $u_2 = +1\ ms^{-1}$, $u_3 = 0$
Just after (see Q5): $v_2 = \frac{(m_2 - m_3)u_2 + 2m_3 u_3}{m_2 + m_3}$

$= \frac{(m-2m) + 4m \times 0}{3m} = -\frac{1}{3}\ ms^{-1}$

$v_3 = \frac{(m_3 - m_2)u_3 + 2m_2 u_2}{m_2 + m_3} = \frac{(2m-m)\times 0 + 2m}{3m} = \frac{2}{3}\ ms^{-1}$

(3) 2 with 1 Just before: $u_1 = 0$, $u_2 = -\frac{1}{3}\ ms^{-1}$
Just after: $v_1 = -\frac{1}{3}\ ms^{-1}$, $v_2 = 0$

i.e. at t time after 1st impact



8 (i) N2 (down is +ve): $m\frac{dv}{dt} = mg - D$

At terminal vel $\frac{dv}{dt} = 0$ & $D = k_d v_t^n$

$\therefore mg - k_d v_t^n \rightarrow v_t = \left(\frac{mg}{k_d}\right)^{1/2}$

(ii) $v_t = \left(\frac{80 \times 9.81}{0.25}\right)^{1/2} = 56.0\ ms^{-1}$

(iii) $k_d v_t = mg \Rightarrow k_d = \frac{80 \times 9.81}{5} = 157\ kg\,s^{-1}$

(iv) $D = \frac{1}{2}C_d \rho_f A v^2 \rightarrow k_d = \frac{1}{2}C_d \rho_f A$

Sphere: $A = \pi a^2$, $m = \frac{4}{3}\pi a^3 \rho_s$ ← density of sphere

$v_t = \left(\frac{mg}{k_d}\right)^{1/2} = \left(\frac{\frac{4}{3}\pi a^3 \rho_s g}{\frac{1}{2}C_d \rho_f \pi a^2}\right)^{1/2} = \left(\frac{8g\rho_s a}{3C_d \rho_f}\right)^{1/2} \propto a^{1/2}$

Using given data: $v_t = \left(\frac{8 \times 9.81 \times 7.85 \times 10^3 \times 0.02}{3 \times 0.5 \times 1.25}\right)^{1/2} = 81.1\ ms^{-1}$