Mechanics — M. Coppins
First Year Physics — 2.11.07

Problem Sheet 3: Lectures 2.5 and 2.6

Exercises

1. A ball of mass 0.2 kg and initial velocity 5.0 m s^{-1} in the $+x$ direction has a one-dimensional elestic collision with a ball of mass 0.3 kg which is also moving in the $+x$ direction, but at 2.0 m s^{-1}. Find the velocities of the two balls after the collision.

2. (i) In Lecture 2.5 we showed that if the mass and velocity of a rocket + its unburnt fuel change by dm and dv (where $dm < 0$), then the total momentum change is $dp = mdv + v_e dm$, where v_e is the exhaust velocity (the speed of the burnt fuel relative to the rocket). Show that if there is an external force F opposing the motion then the acceleration of the rocket is given by

$$\frac{dv}{dt} = \frac{(F_T - F)}{m}$$

where $F_T = \text{thrust} = -v_e \frac{dm}{dt}$. [Note: dm/dt = rate at which fuel is used up < 0.]

   (ii) Launch from the surface of the Earth requires an upward thrust which exceeds the downward weight of the rocket + fuel. A fully fuelled Saturn V rocket has a mass of 3.04×10^6 kg and an exhaust velocity of 2.65×10^3 m s^{-1}. Calculate the minimum value of $|dm/dt|$ needed to get it off the ground.

3. An object of mass 0.5 kg, initially at rest, is pushed with a constant horizontal force of 2.0 N across a horizontal surface. The coefficient of kinetic friction between the object and the surface is 0.3. Calculate:

   (i) the distance moved in 1 s,

   (ii) the work done on the object by the person pushing, in 1 s,

   (iii) the work done on the object by friction, in 1 s,

   (iv) the objects kinetic energy after 1 s,

   (v) the average power supplied by the person pushing, over the 1 s interval, and

   (vi) the average power supplied by friction, over the 1 s interval.

4. A 5 kg block is pushed with a force of 30 N in the $+x$ direction. It is on a surface for which $\mu_s = 0.7$ and $\mu_k = 0.5$.

   (i) What are the forces on the block if it is at rest?

   (ii) What is the acceleration of the block if it is moving?

Continued overleaf

Problems

5. Two particles (masses m_A and m_B) have a 1-D eleastic collision. Before the collision their velocities are u_A and u_B. Show that their velocities after the collision are

$$v_A = \frac{(m_A - m_B)u_A + 2m_B u_B}{m_A + m_B}$$
$$v_B = \frac{(m_B - m_A)u_B + 2m_A u_A}{m_A + m_B} .$$

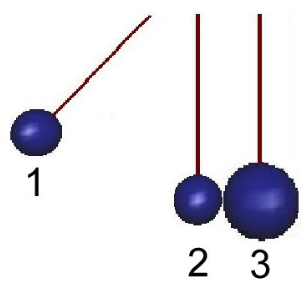
6. A *completely inelastic* collision is one in which the two objects stick together after the collision. In such a collision the total kinetic energy decreases. Consider a completely inelastic collision between two particles (masses m_A and m_B) moving with velocities u_A and u_B before the collision. Show that the change in total kinetic energy can be written

$$\Delta K = -\frac{1}{2}\mu U^2$$

where $\mu = \dfrac{m_A m_B}{m_A + m_B}$ and $U = u_A - u_B$.

Where does the kinetic energy go?

7. An interesting variant on Newton's cradle has three balls. Numbers 1 and 2 have mass m, but number 3 has mass $2m$. Initially numbers 2 and 3 are at rest. Number 1 is released and hits number 2 with a velocity of 1 m s^{-1}. Ignoring losses due to friction and air resistance and assuming all collisions are elastic what are the velocities of the three balls a very short time after the first impact?



8. (i) The drag force which opposes motion through a fluid can be written $D = k_d v^n$ where $n = 1$ for low speeds and 2 for high speeds. Show that the terminal velocity of an object of mass m is $v_t = (mg/k_d)^{1/n}$.

(ii) An 80 kg person falling through the air in a spread-eagle position has $D \propto v^2$ and $k_d = 0.25$ kg m^{-1}. Calculate their terminal velocity.

(iii) With a parachute the 80 kg person's terminal velocity falls to 5.0 m s^{-1}. Assuming $D \propto v$ in this case, calculate the new value of k_d.

(iv) At high speeds the drag force can be written $D = C_d \rho_f A v^2/2$, where C_d is a dimensionless number called the drag coefficient, ρ_f is the density of the fluid, and A is the cross-sectional area of the moving object perpendicular to its motion. Show that the terminal velocity of a falling sphere of radius a is proportional to $\sqrt{a}$, and use the following data to calculate the terminal velocity of a steel sphere of radius 2.0 cm falling in air.

[Density of air = 1.25 kg m^{-3}.

Density of steel = 7.85×10^3 kg m^{-3}.

Drag coefficient for a smooth sphere = 0.5.]

Numerical Answers

1. 1.4 m s^{-1}, 4.4 m s^{-1}
2. (ii) 1.13×10^4 kg s^{-1}
3. (i) 0.53 m, (ii) 1.06 J, (iii) -0.78 J, (iv) 0.28 J, (v) 1.06 W, (vi) -0.78 W
4. (i) 49.0 N downwards, 49.0 N upwards, 30 N in $+x$ direction, 30 N in $-x$ direction. (ii) 1.09 m s^{-2} in $+x$ direction.

7. $v_1 = -0.333$ m s^{-1}, $v_2 = 0$, $v_3 = 0.667$ m s^{-1}

8. (ii) 56.0 m s^{-1}, (iii) 157 kg s^{-1}, (iv) 81.1 m s^{-1}