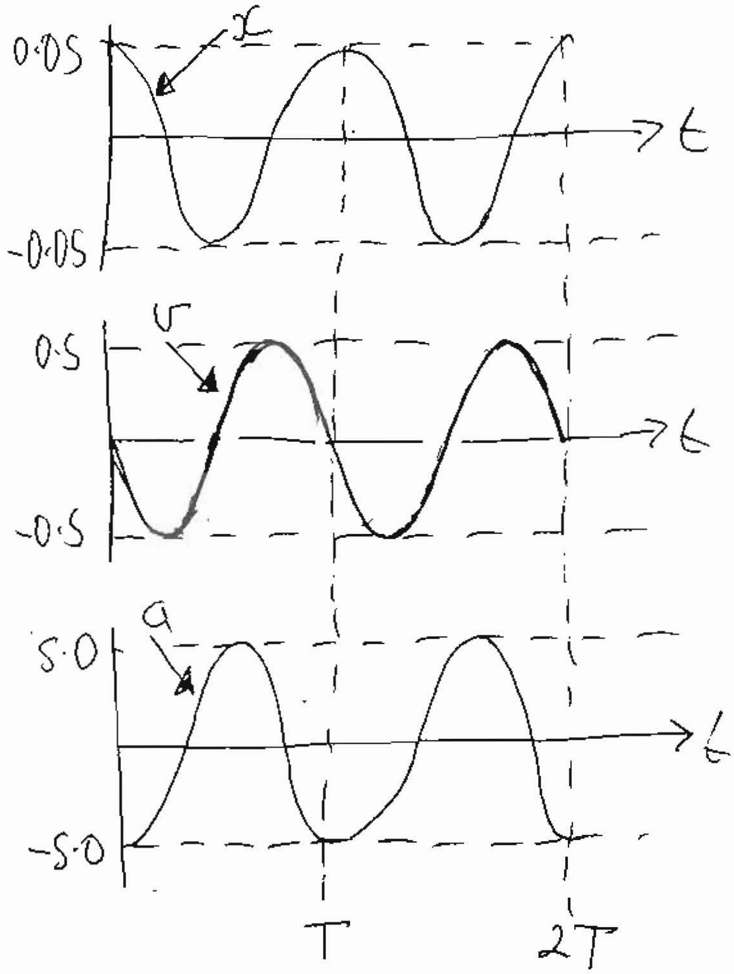
Mechanics, Problem Sheet 2, Answers

1. (i) $x = 0.05\cos(10t) \rightarrow v = -0.5\sin(10t)$, $a = -5.0\cos(10t)$

$v_{max} = 0.5\ ms^{-1}$, $a_{max} = 5.0\ ms^{-1}$

v_{max} when $\sin(10t) = -1 \rightarrow 10t = \frac{3\pi}{2} \rightarrow t = \frac{3\pi}{20} = 0.47s$

a_{max} when $\cos(10t) = -1 \rightarrow 10t = \pi \rightarrow t = \frac{\pi}{10} = 0.31s$



2 $T = 1.0s$, $m = 0.5kg$

$\omega = 2\pi/T = 6.28\ rad\,s^{-1}$, $k = m\omega^2 = 0.5\times(6.28)^2 = 19.7\ Nm^{-1}$

$K_{max} = \frac{1}{2}kx_0^2 = \frac{1}{2}\times 19.7\times(0.02)^2 = 3.95\times10^{-3}\ J$

3. $K_{fin} + U_{fin} = K_{init} + U_{init}$

$\therefore \frac{1}{2}(m_1+m_2)v^2 + m_1gh = 0 + m_2gh$

$$\therefore v^2 = \frac{2(m_2-m_1)gh}{m_1+m_2} = \frac{2\times2\times9.81\times5}{8} \rightarrow v = 4.95\,ms^{-1}$$

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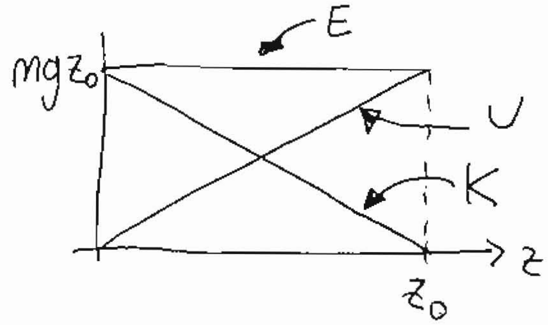

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5 z = vertical displacement of piston = $0.1\cos(\omega t)$
accel = $-0.1\omega^2\cos(\omega t)$
Max. downward accel of piston occurs at the instant the piston reaches its maximum vertical displacement. $\rightarrow a_{max} = -0.1\omega^2$
If $0.1\omega^2 > g$ then coin falls from the max. vertical displacement more slowly than the piston moves down $\rightarrow \omega_{min} = \sqrt{g/0.1} = 9.90\ rad\,s^{-1}$
$\therefore f = \omega/2\pi = 1.58\ Hz$

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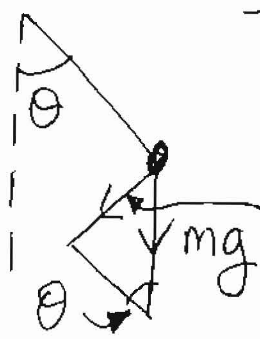
6 (i)

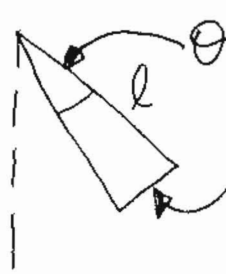


The forces acting on the bob are its weight mg and the tension in the string. Only the weight has a comp ⊥ to the string

mg sin θ in − tangential dirn.

i.e. $F_{tan} = -mg\sin\theta$

(ii)



ds = distance moved (in dt) = $l\,d\theta$

$\therefore v = \frac{ds}{dt} = l\frac{d\theta}{dt}$

$\therefore \frac{dv}{dt} = l\frac{d^2\theta}{dt^2}$. This is accel in tangential dir.

$\therefore m\frac{dv}{dt} = F_{tan} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$ [from (i)]

(iii) $\sin\theta \to \theta$, then $\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$ i.e. SHM eq for θ with $\omega = \sqrt{g/l}$

(iv) $T = 2s \Rightarrow \omega = \pi\ \text{rad s}^{-1} \Rightarrow l = g/\pi^2 = 0.994\ m$

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7 (i) $F = -\frac{dU}{dx} = -\frac{Q^2 a}{2\pi\epsilon_0}\frac{d}{dx}(a^2-x^2)^{-1} = -\frac{Q^2 a}{2\pi\epsilon_0}\left\{-(a^2-x^2)^{-2}(-2x)\right\}$

$= -\frac{Q^2 a \cancel{2}x}{\cancel{2}\pi\epsilon_0}(a^2-x^2)^{-2} = \frac{-Q^2 a x}{\pi\epsilon_0(a^2-x^2)^2}$

(ii) For $|x| \ll a$: $\frac{1}{(a^2-x^2)^2} \simeq \frac{1}{a^4}$ $\therefore F \simeq \frac{-Q^2 a x}{\pi\epsilon_0 a^{\cancel{4}3}}$

i.e. $F \simeq -kx$ with $k = Q^2/\pi\epsilon_0 a^3$

(iii) Lec 2.2: H's law $\Rightarrow$ SHM with $\omega = \sqrt{k/m}$

i.e. charge executes SHM with $\omega = \sqrt{\frac{Q^2}{\pi\epsilon_0 a^3 m}}$

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8 (i) $F = -\frac{dU}{dx} = -x^2 + 1$

(ii) In eqm $F = 0$ $\therefore -x^2+1 = 0$ $\therefore x^2 = 1$ $\therefore x = \pm 1$

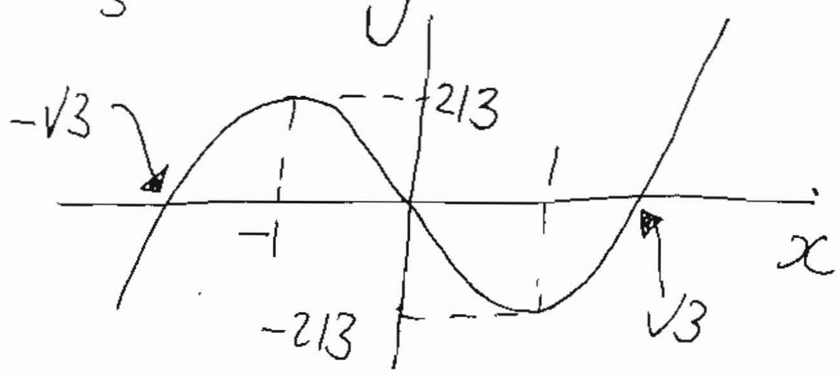
$\frac{d^2U}{dx^2} = \frac{d}{dx}(x^2-1) = 2x$

At $x = -1$: $\frac{d^2U}{dx^2} = -2 \Rightarrow$ max of $U \Rightarrow$ top of potn hill $\Rightarrow$ UNSTABLE

At $x=+1$: $\frac{d^2U}{dx^2}=2 \Rightarrow$ min of $U \Rightarrow$ bottom of ptn well $\Rightarrow$ STABLE

$U_{max} = U(-1) = 2/3$, $U_{min} = U(1) = -2/3$

$U(x)=0 \Rightarrow \frac{x^3}{3} - x = 0 \Rightarrow x(x^2-3)=0 \Rightarrow x=0$ or $x = \pm\sqrt{3}$



(iii) (a) To escape to $-\infty$ particle needs enough energy to get over ptn max at $x=-1$. It will just do so if $E = U+K = 2/3$. But $U(x=+1) = -2/3$

$\therefore K_{min} = \frac{2}{3} - \left(\frac{-2}{3}\right) = \frac{4}{3}$ $\therefore \frac{1}{2} m v_{min}^2 = \frac{4}{3}$ & $m=1$ $\therefore |v| = \sqrt{\frac{8}{3}}\ ms^{-1}$

$U(x=0)=0$ $\therefore K(x=0) = 2/3$

$\therefore$ at $x=0$: $v^2 = 2/(3\times 1/2) \Rightarrow |v| = \sqrt{\frac{4}{3}}\ ms^{-1}$

(b) Particle will just reach origin if $E=0$

$\therefore K(x=+1) = \frac{2}{3} \Rightarrow |v| = \sqrt{\frac{4}{3}}\ ms^{-1}$