Mechanics
First Year Physics

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Problem Sheet 2: Lectures 2.2–2.4

Exercises

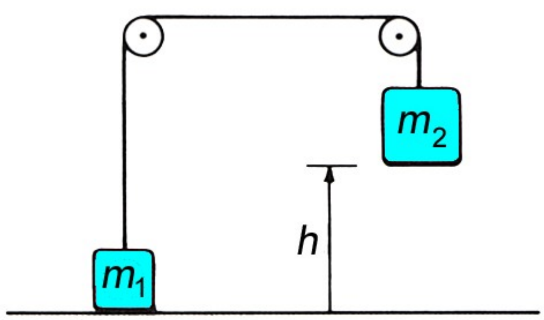
1. An object is executing simple harmonic motion with an angular frequency of 10 rad s^{-1}. At $t = 0$ it is at its maximum displacement of 50 mm.

   (i) Find the maximum (positive) values of the velocity and acceleration, and the first times at which the velocity and acceleration have their maximum (positive) values.

   (ii) Sketch graphs of displacement, velocity, and acceleration for the first two periods.

2. A mass of 0.5 kg oscillates with simple harmonic motion on the end of a spring. The maximum displacement of the spring is 20 mm and the period is 1.0 s. What is the spring constant and the maximum kinetic energy of the mass?

3. Two blocks with masses $m_1 = 3$ kg and $m_2 = 5$ kg are connected by a massless string that slides over two frictionless pegs, as shown below. Initially m_2 is held at height $h = 5$ m from the floor. It is then released. Assuming that air resistance can be neglected, find the the speed with which m_2 hits the floor.



4. An object of mass m falls from rest at height z_0. Assuming air resistance can be neglected, sketch the variation of potential energy, kinetic energy, and total mechanical energy with height above the ground as it falls.

Continued overleaf

Problems

5. A coin rests on the top of a piston which is executing simple harmonic motion in the vertical direction, with a maximum displacement of 10 cm. The frequency is gradually increased. At what frequency does the coin first lose contact with the piston?

6. A simple pendulum is a theoretical idealization of the sort of pendulum you find in a clock. It consists of a mass m (the 'bob') suspended from a fixed point by a massless string of length l.

   (i) In its equilibrium position the bob hangs vertically below the fixed point. Suppose it is displaced through an anticlockwise angle θ. Show that the tangential component of the force (i.e., the component perpendicular to the string) is $F_{tan} = -mg \sin\theta$. [Remenber: the positive tangential direction is the direction of increasing θ.]

   (ii) If the mass moves through a small angle $d\theta$ in time dt show that its speed is given by $v = l\dfrac{d\theta}{dt}$, and, hence, that Newton's second law for motion in the tangential direction is $\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{l}\sin\theta$.

   (iii) The pendulum is given a small angular displacement $\theta = \theta_0$ (such that the approximation $\sin\theta \simeq \theta$ can be used) and released at $t = 0$. Show that the pendulum executes SHM with an angular frequency $\omega = \sqrt{g/l}$.

   (iv) A grandfather clock emitts a 'tick' (or a 'tock') every half-period of its pendulum. Calculate the length of pendulum needed for one tick (or tock) per second.

7. (i) In Lecture 2.4 we analyzed the motion of a charged particle (charge $+Q$) free to move along the x axis and located between two equal $+Q$ charges fixed at $x = -a$ and $x = a$. We found (using a result from electricity and magnetism) that the potential energy was given by:

   $$U = \frac{Q^2 a}{2\pi\epsilon_0(a^2 - x^2)} \ .$$

   Show that the force on the charge is given by:

   $$F = \frac{-Q^2 a x}{\pi\epsilon_0(a^2 - x^2)^2} \ .$$

   (ii) The equilibrium position is $x = 0$. Show that for small displacements from equilibrium ($|x| << a$) the force has the form of Hooke's law $F = -kx$, and find an expression for the constant k.

   (iii) Hence show that for small displacements the charged particle executes simple harmonic motion with an angular frequency given by

   $$\omega = \frac{Q}{\sqrt{\pi\epsilon_0 a^3 m}}$$

   where m is the mass of the charged particle.

8. This is slightly modified version of about 2/3 of an old exam question (2002, Q. 2).

   (i) A particle of mass 1 kg moves along the x axis under the influence of a force for which the potential energy is described by

   $$U(x) = \frac{x^3}{3} - x \ .$$

   Write down an expression for the force on the particle as a function of x.

   (ii) Show that at the points $x = \pm 1$ m the particle is in equilibrium. With appropriate justification, classify each equilibrium point as either stable or unstable. Sketch $U(x)$, labelling the values of x corresponding to zeroes of $U(x)$.

   (iii) The particle is moving in the $-x$ direction at $x = +1$ m. Calculate the speed necessary for the particle to

   (a) just escape to $-\infty$. In this case determine the velocity of the particle as it passes through the origin.

   (b) just to reach the origin.

Numerical Answers

1. (i) 0.5 m s^{-1}, 5.0 m s^{-2}, 0.47 s, 0.31 s
2. 19.7 N m^{-1}, 3.95×10^{-3} J
3. 4.95 m s^{-1}
5. 1.58 Hz
6. (iv) 0.994 m
8. (iii) (a) $\sqrt{8/3}$ m s^{-1}, $\sqrt{4/3}$ m s^{-1}, (b) $\sqrt{4/3}$ m s^{-1}