Mechanics, Problem Sheet 1: Answers

1 (i) $kg\ ms^{-2}$

(ii) $kg\ m^2\ s^{-2}$

(iii) $kg\ m^{-1}\ s^{-2}$

(iv) units of $D = kg\ ms^{-2}$

units of $\rho = kg\ m^{-3}$

units of $A = m^2$

units of $v^2 = m^2 s^{-2}$

$\therefore C_d$ is dimensionless

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2 (i) $V_{av} = \dfrac{50 \times 1.61 \times 10^3}{49 \times 60} = 27.4\ ms^{-1}$

(ii) $v = \dfrac{p}{m} = \dfrac{3.6}{45 \times 10^{-3}} = 80\ ms^{-1}$

(iii) $60\ mph \rightarrow \dfrac{60 \times 1.61 \times 10^3}{60 \times 60} = 26.8\ ms^{-1}$

$\therefore a_{av} = \dfrac{26.8}{7} = 3.83\ ms^{-2}$

(iv) Weight $= 3.04 \times 10^6 \times 9.81 = 2.98 \times 10^7\ N$

$\therefore$ net upward force $= 3.4 \times 10^7 - 2.98 \times 10^7 = 4.2 \times 10^6\ N$

$\therefore a = \dfrac{4.2 \times 10^6}{3.04 \times 10^6} = 1.4\ ms^{-2}$

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3 $F_{elec} = \dfrac{e^2}{4\pi\epsilon_0 r^2}$ $\quad F_{grav} = \dfrac{G m_e^2}{r^2}$

$$\therefore \frac{F_{elec}}{F_{grav}} = \frac{e^2}{4\pi\epsilon_0 G m_e^2} = \frac{(1{\cdot}60\times10^{-19})^2}{4\pi\times8{\cdot}85\times10^{-12}\times6{\cdot}67\times10^{-11}\times(9{\cdot}11\times10^{-31})^2}$$

$= 4{\cdot}16\times10^{42}$ (You don't often find questions on problem sheets with answers this big!)

Both forces are "inverse square" so the separation cancels.

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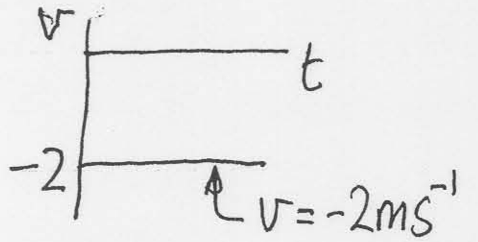
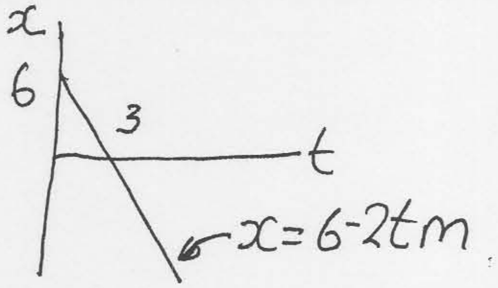
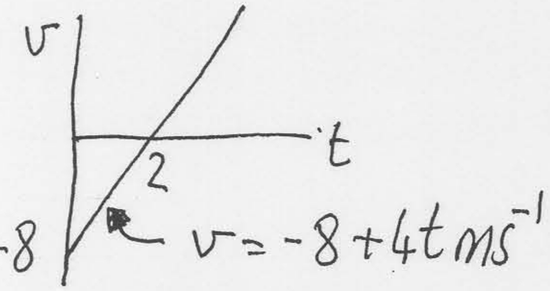
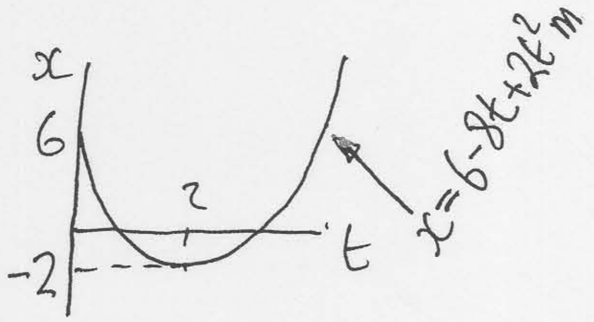
4 (i)



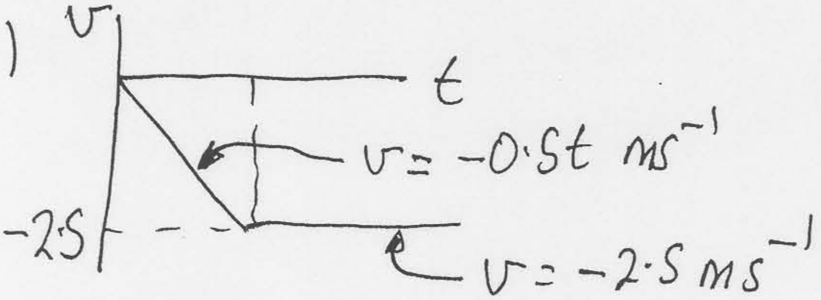


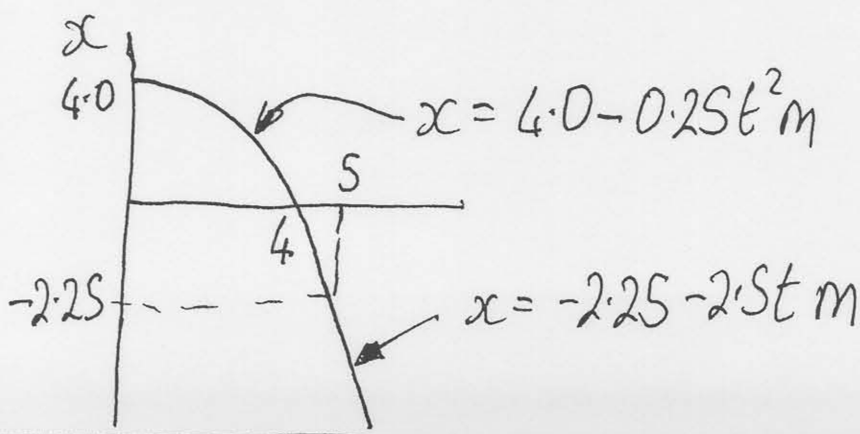
(ii)





(iii)





5 It is true that the horse-cart forces cancel. They are internal to the horse-cart system. To change the total momentum we require an external force, & this is provided by friction with the ground. The horse exerts a force on the ground & the ground exerts an equal & opposite force on the horse, and it is that which moves the horse-cart system. Friction with the ground also acts on the cart and this opposes the motion. So the horse must exert a large enough force that $F_{\text{ground on horse}} > F_{\text{ground on cart}}$.

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6 (i) $\underline{\bar{F}}$ = average force $= \frac{1}{\Delta t}\int_0^{\Delta t} \underline{F}\, dt$

N2: $\frac{d\underline{p}}{dt} = \underline{F}$. Integrate: $\Delta \underline{p} = \int_0^{\Delta t} \underline{F}\, dt = \underline{\bar{F}} \Delta t = \underline{J}$

(ii) Golf ball initially at rest $\therefore \Delta p = 3{\cdot}6\ \text{kg m s}^{-1}$

$\therefore \bar{F} = \frac{\Delta p}{\Delta t} = \frac{3{\cdot}6}{5\times 10^{-3}} = 720\,\text{N}$

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7 (i) $z(t)$ = height at time $t = z_0 + v_0 t + \frac{1}{2}at^2$, $z_0 = h$, $v_0 = 0$, $a = -g$
Time at which $z = 0$ given by $0 = h - \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$

v_f = speed just before hitting ground $= gt = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$

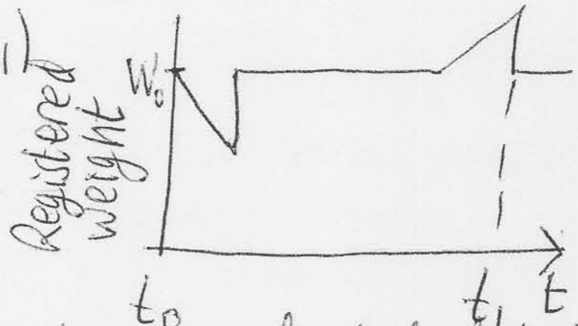
(ii) (a) Time taken for grain to fall height h is $\sqrt{2h/g}$. During this time the no of grains which start to fall $= R \times \text{time} = R\sqrt{2h/g}$ = no of grains in air. $\therefore$ total mass of sand in air $= mR\sqrt{2h/g}$

(b) Each grain imparts mv_f to ground.

F = rate at which mom. imparted $= Rmv_f = mR\sqrt{2gh}$

Weight of sand in air = mass $\times g = mR\sqrt{2gh}$

(iii)



t_0: 1st grain starts to fall

t_1: all sand collected in lower part

W_0 = actual weight of hourglass + sand.

When sand starts to fall & before any reaches the bottom the registered weight is lower (some falling, no impacts). When the top part is empty but some sand still falling the registered weight is higher (less falling, impacts unchanged).

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8 (i) The observer in the lift sees the bolt fall under gravity from rest i.e. $z_0 = 3\,m$, $a = -9.81\,ms^{-2}$

$$z = z_0 + \frac{1}{2}at^2 = 3 - \frac{1}{2}\times 9.81t^2 \rightarrow z = 0 \text{ at } t = \sqrt{\frac{2\times 3}{9.81}} = 0.78s$$

(ii) The observer outside sees the bolt initially moving upwards at $v_0 = 2\,ms^{-1}$. Taking the position of the floor of the lift at the instant the bolt starts to fall as $z = 0$, the outside observer finds:

$$z_{bolt} = z_0 + v_0 t + \frac{1}{2}at^2 = 3 + 2t - \frac{1}{2}\times 9.81t^2$$

But in time t the floor moves to $z_{floor} = 2t$

The time at which the bolt hits the floor given by

$$z_{bolt} = z_{floor} \rightarrow 3 + 2t - \frac{1}{2}\times 9.81t^2 = 2t \rightarrow t = 0.78s \text{ again}$$