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Problem Sheet 1: Lectures 1.1–2.1

Exercises

1. The fundamental mechanical quantities in SI units are length (in metres), mass (in kilogrammes) and time (in seconds). Express the SI unit of each of the following quantities in terms of m, kg, and s.

   (i) force.

   (ii) energy.

   (iii) pressure.

   (iv) The drag coefficient C_d in the equation $D = C_d \rho A v^2/2$ where D is drag force (in N), ρ is density, A is cross-sectional area, and v is the speed.

2. [For some parts of this question you need to use the conversion 1 mile = 1.61 km.]

   (i) A train travels from London to Brighton, a distance of 50 miles, in 49 minutes. Calculate its average speed in m s^{-1}.

   (ii) Just after being hit by Tiger Woods a 45 g golf ball has a momentum of 3.6 kg m s^{-1}. Calculate its speed in m s^{-1}.

   (iii) A car accelerates in a straight line from 0 to 60 miles per hour in 7 s. Calculate its average acceleration in m s^{-2}.

   (iv) A Saturn V rocket has a mass (including fuel) of 3.04×10^6 kg. Its engine exerts a force 34 MN. Calculate its upward vertical acceleration on launch in m s^{-2}. (Assume the mass stays constant.)

3. Calculate the ratio of the electrical force between two electrons to the gravitional force between them. Why haven't you been told their separation?

4. Sketch graphs of v and x against t in each of the following situations (x_0 is the intial position, v_0 is the initial velocity, a is the acceleration).

   (i) $x_0 = 6$ m, $v_0 = -2$ m s^{-1}, $a = 0$,

   (ii) $x_0 = 6$ m, $v_0 = -8$ m s^{-1}, $a = 4$ m s^{-2},

   (iii) $x_0 = 4.0$ m, $v_0 = 0$, $a = -0.5$ m s^{-2} for $t = 0$ to 5 s and zero thereafter.

Problems

5. A horse is required to pull a cart. Unfortunately, the horse chosen for the task not only talks but also has a degree in physics. He refuses to cooperate, arguing that by Newton's third law the force he exerts on the cart will be exactly balanced by the equal and opposite force that the cart exerts on him. Thus, any attempt to pull the cart would be futile. How would you explain the fallacy of this argument to the horse?

6. An impulsive force is a large force which acts for a short time. If the average force applied over a time Δt is $\overline{\mathbf{F}}$, the *impulse* is defined as $\mathbf{J} = \overline{\mathbf{F}}\Delta t$.

   (i) Show that the change in momentum of an object accelerated by an impulse is $\Delta\mathbf{p} = \mathbf{J}$.

   (ii) Calculate the average force exerted by Tiger Woods on the golf ball discussed in Q. 2 (ii) assuming that the club is in contact with the ball for 5 ms.

7. This question involves objects in gravitational free fall. You may neglect air resistance, and assume that gravity is the only force acting on the object while it is falling.

   (i) An object of mass m falls from rest through a vertical height h. Find expressions for the time taken to fall distance h, and the objects speed just before hitting the ground, in terms of h and g, the acceleration due to gravity.

   (ii) Grains of sand, each one of mass m, fall from rest in a steady stream through a vertical height h, at a rate of R grains per second.

      (a) Show that the total mass of sand which is falling through the air at any time is $mR\sqrt{2h/g}$. (You may neglect the height of the sand-pile on the ground compared with h.)

      (b) On impact the sand exerts a force on the ground. By considering the rate at which the sand loses momentum to the ground show that there is a downward force of $mR\sqrt{2gh}$, and, hence, show that the downward force exerted by the sand on the ground is equal to the weight of sand in the air.

   (iii) An inverted hourglass is placed on a weighing scale. ("Inverted" here means with all the sand in the upper part.) Sketch a graph of the weight registered as a function of time, from the moment the sand starts to fall (assume the hourglass was placed on the scale just before this happened) until all the sand has collected in the lower part.

8. A lift of height 3 m is travelling upwards at a speed of 2 m s^{-1}. A bolt in the roof of the lift works free and falls down. Ignoring air resistance, calculate the time taken for the bolt to fall from the roof to the floor of the lift,

   (i) from the point of view of an observer in the lift.

   (ii) from the point of view of a stationary observer outside the lift.

Numerical Answers

2. (i) 27.4 m s^{-1}, (ii) 80 m s^{-1}, (iii) 3.83 m s^{-2}, (iv) 1.4 m s^{-2}.

3. 4.16×10^{42}.

6. (ii) 720 N.

8. 0.78 s.