Mechanics | M. Coppins
First Year Physics | 9.10.07

Handout 1: Course Guidebook

Contents

1. Introduction

1.1 What is it about, and why should you care?

Classical mechanics explains the motion of things. All sorts of things: planets, stars, trains, golf balls, skaters, gyroscopes, things that fall, and things that spin, and things that collide, and things that bounce around on the ends of springs...everything you see in the universe around you. It is the most widely used part of physics.

Of course, it is also quite an old part of physics. It was formulated in the seventeeth century mainly by Galileo and Newton, and since then more accurate theories have been developed. Relativity has superseded classical mechanics for things moving very fast, and quantum theory has superseded it for sub-atomic things. So isn't it time to ditch it from the curiculum, and just teach modern physics?

There are three reasons why you should still study classical mechanics in the 21st century. Firstly, in the history of human culture it is arguably mankinds greatest intellectual achievement. As a physicist you ought to have some appreciation of that achievement. Secondly, it is much easier to learn about many of the basic concepts (like potential energy, or angular momentum) in the context of the 'everyday' world of classical mechanics, and then later apply then in the less familiar (not to say outright bizarre) worlds of quantum physics, etc. Thirdly, it is the most widely used part of physics. Quantum theory won't get a spacecraft to Mars, or help you understand the physics of golf.

1.2 What do you need to know?

This course assumes you know nothing about mechanics. It does assume familiarity with core A-level maths, and in the later parts of the course vectors are used a lot. If you are not familiar with vectors, don't worry because they will have been covered in Maths by the time we get to them.

If you have already done a lot of mechanics in A-level maths, you'll be going over quite a lot of old ground. You will encounter new things, but, more importantly, you'll encounter new ways of looking at things. You'll be studying the material in a more mature, university-level way. So you should still get plenty out of the course.

2. Course structure and arrangements

2.1 Lectures

The course is divided into 5 parts, each comprising a number of lectures. Each lecture is self-contained with a number and a title. Within the lectures there are numbered sections and equations, making it easy to find your way round. The course is also completely self-consistent, in that any result or equation used in one lecture will have been proved in an earlier one. The lectures will include quizzes and opportunities to discuss the material being taught.

The numbers, titles and dates of the lectures are:

Part 1 Foundations
1.1 Basic concepts (9 Oct)
1.2 Newton's laws (11 Oct)
1.3 Forces (16 Oct)
Part 2 1-D motion
2.1 1-D motion with constant acceleration (18 Oct)
2.2 Simple harmonic motion (22 Oct)
2.3 Potential energy (25 Oct)
2.4 Energy and forces (26 Oct)
2.5 Collisions and rockets (1 Nov)
2.6 Dissipative forces (2 Nov)
Part 3 3-D motion
3.1 Vectors in mechanics (8 Nov)
3.2 Frames and pseudo-forces (11 Nov)
3.3 Energy and forces in 3-D (13 Nov)
3.4 Centre of mass (15 Nov)
3.5 3-D kinematics (16 Nov)
3.6 Angular momentum (20 Nov)
Part 4 Rigid bodies
4.1 Rotation about a fixed axis (23 Nov)
4.2 Moments of inertia (27 Nov)
4.3 General motion of a rigid body (30 Nov)
4.4 Skating and gyroscopes (4 Dec)
Part 5 Gravity
5.1 Central forces (7 Dec)
5.2 Orbits (11 Dec)
5.3 Tidal forces (14 Dec)

2.2 Problem sheets

In addition to the assessed problem sheets (see below) there will be 9 regular problem sheets, one per week from Week 3 onwards. Each problem sheet will have two parts: a set of 4 exercises (fairly straightforward questions, usually involving plugging numbers into equations), and 4 problems (more challenging problems, designed to get you thinking about the course material).

2.3 Classworks

There will be 6 classworks:
Classwork I Gravity, weight and tidal forces (25 Oct)
Classwork II Ion thrusters (1 Nov)
Classwork III The Punkin Chunkin World Championships (8 Nov)
Classwork IV The coefficient of restitution (15 Nov)
Classwork V Racing cylinders down slopes (29 Nov)
Classwork VI Gravity and the pseudo-potential (14 Dec)

2.4 Assessment

Questions on this course will be included in the end-of-year Mechanics and Relativity exam, and in the New Year Test in January. There will also be questions on mechanics on the assessed problem sheets in term 1 (26 Oct, 9 and 23 Nov).

2.5 Books

The material in this course is covered very comprehensively in Young and Freedman, *University Physics* (11th edition), chapters 2-13. This book also provides a good source of supplementary problems.

As a stand-alone introductory level textbook on mechanics I recommend *Classical Mechanics. A modern introduction* by Martin McCall.

2.6 If you have difficulties

Office hours. Mondays and Thursdays, 1-2, in Room 726 Blackett. Please don't hesitate to come and see me.

Email. Feel free to email me (m.coppins@imperial.ac.uk) with questions, comments, or suggestions for improving the course.

3. Symbols used in the lectures

Symbols in bold represent vectors. When hand-written they are underlined. The SI unit of each variable is given in brackets.

A: area (m^2).
$\mathbf{a}$: acceleration (m s^{-2}).
$\mathbf{D}$: drag force (N).
E: total mechanical energy (J).
$\mathbf{F}$: force (N).
f: frequency (s^{-1}).
$\mathbf{f}$: friction ($\mathbf{f}_s$, $\mathbf{f}_k$: static and kinetic friction) (N).
G: gravitational constant $= 6.67 \times 10^{-11}$ N m^2 kg^{-2}.
$\mathbf{g}$: acceleration due to gravity close to the Earth's surface $= 9.81$ m s^{-2}.
I: moment of inertia (kg m^2).
$\mathbf{J}$: impulse (kg m s^{-1}).
K: kinetic energy (J).
k: spring constant (N m^{-1}).
$\mathbf{L}$: angular momentum (kg m^2 s^{-1}).
l: length, specifically, length of a spring, and lever arm (m).
M, m: mass (kg).

$\mathbf{n}$: normal contact force (N).
P: power (W or J s^{-1}).
$\mathbf{p}$: momentum (kg m s^{-1}).
Q, q: charge (C).
$\mathbf{r}$: position vector (m).
$\mathbf{r}_{CM}$: position vector of centre of mass (m).
$\mathbf{T}$: tension (N).
T: period (s).
$\mathbf{U}$: relative velocity (m s^{-1}).
U: potential energy (J).
U_{int}: internal energy (J).
$\mathbf{u}$: velocity (m s^{-1}).
V: volume (m^3).
$\mathbf{v}$: velocity (m s^{-1}).
$\mathbf{v}_{CM}$: velocity of centre of mass (m s^{-1}).
W: work done (J).
$\boldsymbol{\alpha}$: angular acceleration (rad s^{-2}).
μ: reduced mass (kg).
μ_s, μ_k: coefficient of static/kinetic friction (dimensionless).
$\boldsymbol{\tau}$: torque (N m).
Ω: precessional angular speed (rad s^{-1}).
$\boldsymbol{\omega}$: angular velocity (rad s^{-1}).
ω: angular frequency or angular speed (rad s^{-1}).

4. Summary

This is a list of the equations, definitions, facts and laws you *need to memorize* for the Mechanics exam. It is not as overwhelming as it looks. Because it is divided into the 5 parts, some results appear more than once (the work-energy theorem, for instance, turns up in both Part 2 and Part 3). Furthermore, you probably already know a lot of these things (for instance, the definition of kinetic energy).

On the other hand, there are a lot of other things, not on this list, which you must understand, for instance, how to use a free-body diagram to solve for components of forces, or why a gyroscope precesses. And there are many other equations in the course which, even though you're not expected to memorize them exactly, you should be able to derive: expressions for the moment of inertia of a hollow cylinder or the period of a circular orbit around the Earth, for instance.

Each item in the list has a reference to enable you to find it in the notes. Mostly these are equation numbers (for instance, 2.1.4.3 is the 3rd numbered equation in Section 2.1.4, which itself is the 4th section of lecture 2.1). Occasionally there are references to problem sheets, and one item is a whole section of lecture 4.3. Newton's laws (of motion) are awarded special status in the course, and in the notes are simply referred to as N1, N2, and N3.

Part 1: Foundations

1.1.4.1. $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$.

1.1.4.2. Magnitude of $\mathbf{v}$ is speed.

1.1.4.3. Direction of $\mathbf{v}$ at any instant is the direction of motion at that instant.

1.1.4.4. $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$.

1.2.1.1. Principle of superposition. If several forces act simultaneously, the net (resultant) force is the vector sum of the separate forces.

Newton's First Law. A body with no net force acting on it moves at a constant velocity.

Newton's Second Law. A body with a net force acting on it accelerates, and $\mathbf{F} = \dfrac{d\mathbf{p}}{dt}$.

Newton's Third Law. If body A exerts a force on body B, then body B exerts an equal and opposite force on body A.

1.2.4.1. Inertia is the tendency of an object to resist changes of motion.

1.2.4.2. $\mathbf{p} = m\mathbf{v}$.

1.2.5.1. Forces in an action-reaction pair never act on the same object.

1.2.6.1. $\dfrac{d\mathbf{p}_{tot}}{dt} = \sum \mathbf{F}^{ext}$.

1.2.6.2. If $\mathbf{F}^{ext} = 0$ then $\mathbf{p}_{tot} =$ constant (conservation of momentum).

1.3.2.1. Newton's law of universal gravitation. There is an attractive force between any two particles (masses m_1 and m_2, distance r apart) of magnitude $F = \dfrac{Gm_1m_2}{r^2}$.

1.3.5.1. Weight is the gravitational force on an object from everything else in the universe.

Problem Sheet 1. $\mathbf{J} = \overline{\mathbf{F}}\Delta t$.

Problem Sheet 1. $\Delta\mathbf{p} = \mathbf{J}$.

Part 2: 1-D motion

2.1.2.1. For motion with a constant acceleration: $v = v_0 + at$.

2.1.2.2. For motion with a constant acceleration: $x = x_0 + v_0 t + \dfrac{1}{2}at^2$.

2.1.4.2. Work done is force $\times$ distance moved.

2.1.4.3. Kinetic energy $= K = \dfrac{1}{2}mv^2 =$ energy associated with an objects speed.

2.1.4.4. Work-energy theorem: $W = \Delta K$.

2.1.5.1. In free fall: $v = -gt$.

2.1.5.2. In free fall from height h: $z = h - \dfrac{1}{2}gt^2$.

2.2.2.1. Hooke's law. For small displacements from equilibrium (e.g., of a spring): $F = -kx$.

2.2.5.1. Simple harmonic motion (SHM) equation: $\dfrac{d^2x}{dt^2} = -\omega^2 x$.

2.2.3.2. In SHM: $x(t) = x_0 \cos(\omega t)$[1]

2.2.3.4. In SHM: $T = \dfrac{2\pi}{\omega}$.

2.3.0.1. Potential energy is energy associated with an objects position.

2.3.1.2. $E = K + U$.

2.3.1.1. For gravity (close to Earth's surface): $U = mgz$.

2.3.2.1. For systems described by Hooke's law (e.g., springs): $U = \dfrac{1}{2}kx^2$.

2.3.3.1. $P = \dfrac{dW}{dt}$.

2.3.3.2. $P = Fv$.

2.3.4.1. Conservative force = force for which the total mechanical energy is constant.

2.3.4.2. For a conservative force: $W = -\Delta U$.

2.3.4.3. For a conservative force: $F = -\dfrac{dU}{dx}$.

2.4.1.1. The bottom of a potential well is a point of stable equilibrium.

2.4.1.2. The top of a potential hill is a point of unstable equilibrium.

2.5.1.1. Elastic collision: the total kinetic energy remains constant.

2.5.1.2. Inelastic collision: some kinetic energy is lost.

2.6.0.1. Dissipative force = non-conservative force which reduces E.

Part 3: 3-D motion

3.1.1.1. $\mathbf{v}_{BA}$ = velocity of B relative to A = $\mathbf{v}_B - \mathbf{v}_A$.

3.1.1.3. $\mathbf{v}' = \mathbf{v} - \mathbf{u}$, where $\mathbf{v}'$ = velocity in frame 2, $\mathbf{v}$ = velocity in frame 1, and $\mathbf{u}$ = velocity of frame 2 with respect to frame 1.

3.1.3.2. Projectiles have horizontal motion at constant velocity.

3.1.3.4. Projectiles have vertical motion at constant acceleration.

3.1.3.6. Projectiles follow a parabolic trajectory.

3.2.1.2. Inerial frames are stationary or moving at constant velocity.

3.2.1.3. Newtons laws are valid in all inertial frames.

3.2.3.1. To use Newton's 2nd law in a non-inertial frame, which has an acceleration $\mathbf{a}$, we must include an extra pseudo-force with the properties: (i) the magnitude of the force on mass m is ma, and, (ii) the direction of the force is opposite to the direction of $\mathbf{a}$.

3.2.5.1. An observer in gravitational free fall experiences weightlessness.

3.3.1.2. $dW = \mathbf{F}.d\mathbf{l}$.

3.3.1.3. $W = \displaystyle\int \mathbf{F}.d\mathbf{l}$.

3.3.2.1. $P = \dfrac{dW}{dt} = \mathbf{F}.\mathbf{v}$.

[1] In general the solution of the SHM equation is a combination of cos's and sin's.

3.3.2.2. Work-energy theorem: $W = \Delta K$.

3.3.3.1. For a conservative force: $W = -\Delta U$.

3.3.3.2. For a conservative force: $\mathbf{F} = -\nabla U$.

3.4.1.1. $\mathbf{r}_{CM} = \dfrac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$.

3.4.2.1. $M\mathbf{v}_{CM} = \mathbf{p}_{tot}$.

3.4.2.2. $M\dfrac{d\mathbf{v}_{CM}}{dt} = \sum \mathbf{F}^{ext}$.

3.4.2.3. The centre of mass moves as if the total mass were concentrated there and only external forces acted.

3.4.2.4. If $\mathbf{F}^{ext} = 0$ then $\mathbf{v}_{CM}$ is constant.

3.4.3.1. The total momentum in the centre-of-mass frame is zero.

3.5.1.1. The acceleration of a particle following a curved path is directed towards the inside of the curve.

3.5.1.2. In general $\mathbf{a}$ has parallel and perpendicular (to $\mathbf{v}$) components: $\mathbf{a}_{\|}$ changes the speed, and $\mathbf{a}_{\perp}$ changes the direction.

3.5.2.2. $\omega = \dfrac{d\theta}{dt}$.

3.5.2.3. In circular motion: $\omega = \dfrac{v}{r}$

3.5.2.4. The direction of $\boldsymbol{\omega}$ is perpendicualr to the plane of motion, given by the right-hand rule.

3.5.2.5. $\boldsymbol{\alpha} = \dfrac{d\boldsymbol{\omega}}{dt}$.

3.5.2.6. In uniform circular motion: $\boldsymbol{\alpha} = 0$.

3.5.2.7. In uniform circular motion: $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$.

3.5.3.3. In polar coordinates: $\mathbf{v} = \dfrac{dr}{dt}\hat{\mathbf{r}} + \omega r\hat{\boldsymbol{\theta}}$.

3.5.4.3. In uniform circular motion the acceleration points radially inwards (centripetal acceleration).

3.5.4.4. In uniform circular motion the acceleration has magnitude $= \omega^2 r = \dfrac{v^2}{r}$.

3.6.1.1. $\mathbf{L} = \mathbf{r} \times \mathbf{p}$.

3.6.1.2. $L = mrv_\theta$.

3.6.2.1. $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.

3.6.2.2. $\dfrac{d\mathbf{L}}{dt} = \boldsymbol{\tau}$.

3.6.3.1. $\dfrac{d\mathbf{L}_{tot}}{dt} = \sum \boldsymbol{\tau}^{ext}$.

3.6.3.2. If $\boldsymbol{\tau}^{ext} = 0$ then $\mathbf{L}_{tot} =$ constant (conservation of angular momentum).

3.6.4.1. In circular motion $W = \displaystyle\int \tau d\theta$.

Part 4: Rigid bodies

4.1.2.1. All points in a rigid body have the same $\boldsymbol{\omega}$, and the direction of $\boldsymbol{\omega}$ is along the axis of rotation.

4.1.3.1. $I = \sum_i m_i \tilde{r}_i^2$, where $\tilde{r}_i$ is the distance of particle i from the axis.

4.1.3.2. $K = \frac{1}{2} I\omega^2$.

4.1.4.2. Parallel axis theorem. $I_p = I_{CM} + Md^2$, where I_{CM} is the moment of inertia about an axis through the centre of mass, and I_p is the moment of inertia about a parallel axis distance d away.

4.2.4.3. For a thin cylindrical shell (mass M, radius R), the moment of inertia about the symmetry axis is MR^2.

4.3.1.1. We can describe the motion of a rigid body as: (i) translation of a point in the body, plus, (ii) rotation about that point. The angular velocity is independent of the point chosen.

4.3.2.1. We can determine the motion of a rigid body by (i) using 3.4.2.2 to find the translational motion of the centre of mass, and (ii) 3.6.3.1 to find the rotation about the centre of mass.

4.3.3.1. For rotation about an axis of symmetry $\mathbf{L}$ and $\boldsymbol{\omega}$ are parallel, and $\mathbf{L} = I\boldsymbol{\omega}$.

Sec. 4.3.4 The standard mechanical variables and the equivalent variables for rotational motion are summarized in the following table:

Standard variable	Rotational variable
$\mathbf{v}$	$\boldsymbol{\omega}$
$\mathbf{a}$	$\boldsymbol{\alpha}$
m	I
$\mathbf{p} = m\mathbf{v}$	$\mathbf{L} = I\boldsymbol{\omega}$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
$\mathbf{F} = \frac{d\mathbf{p}}{dt}$	$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$

4.4.2.2. The torque exerted by gravity on a rigid body acts as if the total mass were concentrated at the centre of gravity = centre of mass (assuming the acceleration of gravity is uniform over the body).

Part 5: Gravity

5.1.1.1. Newton's law of universal gravitation. The force between any two particles (masses m_1 and m_2, distance r apart) is $\mathbf{F} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$.

5.1.2.1. For any central force the angular momentum about the centre of the force is constant if no other forces act.

5.1.3.1. Gravitational potential energy: $U = -\frac{Gm_1m_2}{r}$.

5.1.4.1. The gravitational potential energy and force outside any spherically symmetric mass distribution is the same as those of a point particle at the centre of the sphere with the same total mass.

5.1.4.2. The gravitational force inside a hollow spherical shell is zero.

5.2.1.1. Body A in gravitational orbit around body B is continually 'falling' towards body B.

5.2.1.2. Objects in gravitational orbit experience weightlessness because they are in free fall.