MECHANICS CLASSWORK VI Answers

(i) $\frac{d\underline{L}}{dt} = \underline{\tau} = \underline{r} \times \underline{F}$ $\underline{r}$ = position vector of planet wrt star

$\underline{F}$ is in dirn of $\underline{r} \Rightarrow \underline{\tau} = 0 \Rightarrow \underline{L} = \text{const}$

(ii)

Use polar coords with star at origin

In time dt line sweeps out angle $d\theta$

dA = area swept out $= \frac{1}{2} r^2 d\theta$

$\therefore \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega$

But $L = mv_\theta r = m\omega r^2 = \text{const}$

$\therefore \omega r^2 = \frac{L}{m} = \text{const}$ $\therefore \frac{dA}{dt} = \frac{L}{2m} = \text{const}$

(iii) Centripetal accel $= v^2/R$

N2: $m\frac{v^2}{R} = \frac{GMm}{R^2}$ ← M = mass of planet

$\therefore v = \left(\frac{GM}{R}\right)^{1/2}$ $T = \frac{2\pi R}{v} = \frac{2\pi R^{3/2}}{(GM)^{1/2}}$

(iv) $R = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$

For Earth $R = \left(\frac{6.67\times10^{-11}\times1.99\times10^{30}\times T^2}{4\pi^2}\right)^{1/3}$

$= 1.50\times10^{11}\,m$ $\quad T = 365.25\times24\times60\times60$

(v) Centripetal accel $= \omega^2 r$

Star 1: $M_1\omega^2 r_1 = \frac{GM_1M_2}{(r_1+r_2)^2}$

Star 2: $M_2\omega^2 r_2 = \frac{GM_1M_2}{(r_1+r_2)^2}$

Add: $\omega^2(r_1+r_2) = \frac{G(M_1+M_2)}{(r_1+r_2)^2} \Rightarrow \omega = \left(\frac{GM_{tot}}{d^3}\right)$ (M_1+M_2; $d = r_1+r_2$ = separation)

$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi d^{3/2}}{(GM_{tot})^{1/2}}$

i.e. eq found in (iii) is OK if $R \to d$ & $M \to M_{tot}$
