MECHANICS CLASSWORK V : ANSWERS

1. Cylinder 1: Solid, wood, $L = 0.1\,m$, $R_1 = 0.05\,m$

$$Vol = \pi R_1^2 L = 7.85 \times 10^{-4} m^3$$
$$\therefore M_1 = 7.85 \times 10^{-4} \times 500 = 0.393\ kg$$
$$I_1 = \frac{1}{2} M_1 R_1^2 = 4.91 \times 10^{-4}\ kg\ m^2$$

Cylinder 2: Solid, Pu, $L = 0.1\,m$, $R_2 = 0.05\,m$

$$Vol = 7.85 \times 10^{-4} m^3$$
$$\therefore M_2 = 7.85 \times 10^{-4} \times 1.98 \times 1.98 \times 10^{4} = 15.6\ kg$$
$$I_2 = \frac{1}{2} M_2 R_2^2 = 1.94 \times 10^{-2}\ kg\ m^2$$

Cylinder 3: Solid, Pu, $L = 0.1\,m$, $M_3 = M_1 = 0.393\,kg$

$$\therefore \rho_{Pu} \pi R_3^2 L = \rho_{wood} \pi R_1^2 L$$
$$\therefore R_3 = R_1 \left(\frac{\rho_{wood}}{\rho_{Pu}}\right)^{1/2} = 7.95 \times 10^{-3}\,m$$
$$I_3 = \frac{1}{2} M_3 R_3^2 = 1.24 \times 10^{-5}\ kg\ m^2$$

Cylinder 4: Hollow, Pu, $L = 0.1\,m$, $R_4 = 0.05\,m$, $M_4 = M_1 = 0.393\,kg$

$$\therefore \rho_{Pu} \pi L (R_4^2 - r_4^2) = \rho_{wood} \pi L R_1^2$$

$R_4 = R_1 = 0.05$

$$\therefore \rho_{Pu} r_4^2 = (0.05)^2 (\rho_{Pu} - \rho_{wood})$$
$$\therefore r_4^2 = (0.05)^2 \left(1 - \frac{\rho_{wood}}{\rho_{Pu}}\right) \quad \therefore r_4 = 4.94 \times 10^{-2}\,m$$
$$I_4 = \frac{1}{2} M_4 (R_4^2 + r_4^2) = 9.70 \times 10^{-4}\ kg\ m^2$$

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2 $K = \underbrace{\frac{1}{2} M v^2}_{(1)} + \underbrace{\frac{1}{2} I \omega^2}_{(2)}$ (1) = ke of CoM
(2) = ke of motion about CoM.

Rolls without slipping

Point of contact moving with speed ωR wrt CoM

But point of contact instantaneously at rest $\therefore v =$ CoM speed $= \omega R$

$\therefore K = \frac{1}{2}Mv^2 + \frac{1}{2}Iv^2/R^2 = \frac{1}{2}Mv^2\left(1 + \frac{I}{MR^2}\right) \quad \beta$

Falls from rest thro' vertical height H. Conservation of total mech energy $\Rightarrow K_{bottom} = \frac{1}{2}Mv_0^2(1+\beta) = MgH \therefore v_0 = \sqrt{\frac{2gH}{1+\beta}}$

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3 For given H, v_0 depends only on β

1, 2 & 3 : all solid cylinders $\Rightarrow \beta = 0.5$

$\therefore v_{01} = v_{02} = v_{03} = \left(\frac{2 \times 9.81 \times 1.0}{1.5}\right)^{\frac{1}{2}} = 3.62\,ms^{-1}$

4 : $\beta = \frac{I_4}{M_4R_4^2} = 0.987 \therefore v_{04} = \left(\frac{2 \times 9.81 \times 1.0}{1.99}\right)^{\frac{1}{2}} = 3.14\,ms^{-1}$

i.e. cylinders 1, 2 & 3 all arrive together, followed by 4

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4 (i) Frozen soup is solid cylinder $\rightarrow \beta = 0.5$.

Liquid soup : tin rotates but if soup doesn't $I \simeq M_{tin}R^2$ where M = total mass = $M_{tin} + M_{soup}$, & assuming tin can be approximated by thin cylindrical shell).

Would expect $M_{soup} > M_{tin} \Rightarrow M_{tin} < M/2$

$\therefore \beta = \frac{M_{tin}R^2}{MR^2} < 0.5$ i.e. lower β than frozen soup.

$\Rightarrow$ expect liquid soup to arrive first

(ii) I thought of the following factors (there are probably many more):

- How much air there is in the tin.
- whether the soup was frozen with the tin upright or on its side (latter $\Rightarrow$ air space at side $\Rightarrow$ cylindrical symmetry destroyed $\Rightarrow$ motion hard to analyze)
- Viscosity of soup (very viscous liquid would acquire spin from rotating tin)
- whether there are solid lumps in soup (they would change mass distribution & affect motion).
