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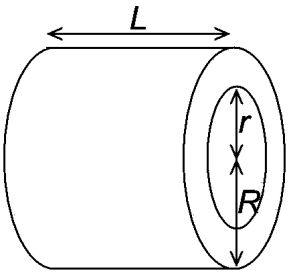
Classwork V
Racing cylinders down a slope

Information needed for this Classwork

Moment of inertia of hollow cylinder, mass M, inner radius r and outer radius R: $I = \frac{1}{2}M(R^2 + r^2)$.



The density of pinewood is 500 kg m^{-3}.

The density of Plutonium is 1.98×10^4 kg m^{-3}.

Potential energy of object mass m height h above the surface of the Earth is mgh.

Acceleration due to gravity: $g = 9.81$ m s^{-2}.

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1. Consider four cylinders, all of length $L = 0.1$ m.

   - Cylinders 1, 2 and 3 are all solid, cylinder 4 is hollow.
   - Cylinders 1, 2 and 4 are of outer radius 0.05 m.
   - Cylinders 1, 3 and 4 all have the same mass.
   - Cylinder 1 is made of pinewood, cylinders 2, 3 and 4 are made of Plutonium.

   Calculate the values of R = outer radius, M = mass, and I = moment of inertia, for the four cylinders, and enter them in the following table.

	R (m)	Mass (kg)	I (kg m^2)
1	0.05		
2	0.05		
3			
4	0.05		

Continued overleaf

2. A cylinder (which may, or may not, be hollow) of outer radius R, mass M, and moment of inertia I, rolls without slipping down a slope. Show that at any instant its kinetic energy is given by: $K = \frac{1}{2}M(1+\beta)v^2$, where v = the speed of the centre of mass, and $\beta = \frac{I}{MR^2}$. Assuming that the cylinder was released from rest at the top of the slope, at vertical height H above the ground, show that the centre of mass speed at the bottom of the slope is given by:

$$v_0 = \sqrt{\frac{2gH}{1+\beta}} \ .$$

3. The four cylinders considered in Question 1 are released simultaneously from the top of a slope at a vertical height of 1.0 m. Assuming that they roll down the slope without slipping, and that air resistance and other dissipative forces are negligible, calculate the centre of mass speeds of each of the cylinders at the bottom of the slope. Hence, determine the order in which they arrive at the bottom.

4. Two identical tins of soup roll, without slipping, down a slope. One of them has been frozen, in the other the soup is still liquid.

   (i) Which one do you think would reach the bottom quickest?

   (ii) What factors do you think affect the relative timings?

Numerical Answers

1.

	R (m)	Mass (kg)	I (kg m^2)
1	0.05	0.393	4.91×10^{-4}
2	0.05	15.6	1.94×10^{-2}
3	7.95×10^{-3}	0.393	1.24×10^{-5}
4	0.05	0.393	9.70×10^{-4}

3. $v_{01} = v_{02} = v_{03} = 3.62$ m s^{-1}, $v_{04} = 3.14$ m s^{-1}