Classwork IV
The coefficient of restitution

Information needed for this Classwork

In an *elastic* collision the total kinetic energy of the colliding objects is conserved.
In an *inelastic* collision some fraction of the kinetic energy of the colliding objects is lost.
In a *completely inelastic* collision the colliding objects stick together after the collsion.

For a collision between two particles, A and B:

- momenta before collsion in centre of mass frame: $\mathbf{p}'_{A\ before} = \mu\mathbf{U}$, $\quad \mathbf{p}'_{B\ before} = -\mu\mathbf{U}$
- momenta after collsion in centre of mass frame: $\mathbf{p}'_{A\ after} = \mu\mathbf{V}$, $\quad \mathbf{p}'_{B\ after} = -\mu\mathbf{V}$

where $\mathbf{U}$ and $\mathbf{V}$ are the relative velocities of the particles before and after the collsion respectively, and $\mu = \dfrac{m_A m_B}{m_A + m_B}$ = the reduced mass.

Potential energy of object mass m height h above the surface of the Earth is mgh.

The height above the ground of an object moving vertically under gravity is given by $z = z_0 + v_0 t - \dfrac{1}{2}gt^2$.

Acceleration due to gravity: $g = 9.81$ m s^{-2}.

The *coefficient of restitution* (COR) is a measure of just how inelastic a collsion is. It basically describes the bounciness of the colliding objects. This is a matter of great importance in ball games. For instance, golfers get very agitated about their COR's (the United States Golf Association has decreed that the COR must not exceed 0.83 in the clubhead-ball impact). This classwork explores a rather surprising method of measuring the COR of a ball by listening to the sound of it bouncing on the ground.

1. Consider a collision between two objects. Show that the total kinetic energy of the two objects in the centre of mass frame is:

   (i) $\dfrac{1}{2}\mu U^2$ before the collision, and

   (ii) $\dfrac{1}{2}\mu V^2$ after the collision,

   where $\mathbf{U}$ and $\mathbf{V}$ are the relative velocities of the particles before and after the collsion respectively.

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2. The COR is defined as $c = |\mathbf{V}|/|\mathbf{U}|$. Show that:

   (i) $c = 1$ for an elastic collsion.

   (ii) $c = 0$ for a completely inelastic collsion.

3. For a ball bouncing on the ground the colliding objects are the ball and the Earth. In this case the centre of mass can be assumed to be fixed at the centre of the Earth, i.e., the "centre of mass frame" and the "lab frame" are the same, and the relative velocity is just the velocity of the ball. Ignoring drag, and assuming that the motion is 1-D (vertical), show that if a ball is dropped from rest from height H the maximum height it will reach after the first bounce from the ground is $h_1 = c^2 H$.

4. Show that the velocity of the ball (upwards):

   (i) just after the 2^{nd} bounce is: $v_2 = c^2\sqrt{2gH}$.

   (ii) just after the n^{th} bounce is: $v_n = c^n\sqrt{2gH}$.

5. Show that the time between the n^{th} and $(n+1)^{th}$ bounce is given by $\Delta t_n = c^n\sqrt{\dfrac{8H}{g}}$.

6. In an experiment to measure the COR, a ball is dropped from a given height, and the sound of its successive bounces on the ground is recorded. From this the time between the n^{th} and $(n+1)^{th}$ bounce, Δt_n, can be measured. In one case it is found that a graph of $T = \ln(\Delta t_n)$ against n has the form of a straight line given by the equation: $T = -0.163n - 0.449$. Find:

   (i) the COR, c, and,

   (ii) the original height from which the ball was dropped, H.

Numerical Answers

6. (i) 0.85, (ii) 0.5 m