Classwork III
The Punkin Chunkin World Championships

Information needed for this Classwork

The trajectory of a particle launched at the origin with a velocity v_0 at an angle θ to the horizontal is given by the equation $z = -Ax^2 + Bx$, where $A = \dfrac{g}{2v_0^2 \cos^2\theta}$ and $B = \tan\theta$.

A projectile moves with a constant velocity in the horizontal direction and a constant acceleration in the vertical direction.

Potential energy of object mass m height h above the surface of the Earth is mgh.

Acceleration due to gravity: $g = 9.81$ m s^{-2}.

$\sin(2\theta) = 2\sin\theta\cos\theta$.

The Punkin Chunkin World Championships are held annually in Dewey Beach, Delaware, USA. Competitors use home-made machines to project pumpkins. The team whose pumpkin lands (intact) furthest from the firing line wins. At the time this Classwork went to press the results for the 2007 competition were not available (it takes place November 2–4), but in 2006 the winners fired their pumpkin from a compressed air canon (the rules forbid the use of explosives) called Second Amendment. The pumpkin landed 1.18 km away. (This is all perfectly true: see www.punkinchunkin.com) In this classwork we analyze the flight of this pumpkin.

For these questions assume the pumpkin can be treated as a particle, which is launched at ground-level, and that air resistance can be neglected. The pumpkin's trajectory is 2-dimensional, with x the horizontal distance and z the vertical distance (upwards defined as positive).

1. A particle is launched at ground-level with a velocity v_0 at an angle θ to the horizontal. Write down expressions for:

   (i) v_{0x} = initial horizontal component of velocity, and

   (ii) v_{0z} = initial vertical component of velocity.

2. By using the conservation of total mechanical energy show that the maximum height reached is
$$H = \frac{v_0^2 \sin^2\theta}{2g} \ .$$

3. By finding the values of x for which $z = 0$ show that the range of the particle (i.e., the horizontal distance it travels before hitting the ground) is given by
$$R = \frac{v_0^2 \sin(2\theta)}{g} \ .$$

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4. Show that the duration of the particles flight is given by

$$T = \frac{2v_0 \sin\theta}{g} \ .$$

5. If $\theta = 0$ (fired horizontally) or $\theta = 90^o$ (fired straight up) the range is zero. Show that for a given v_0 the maximum range is achieved for $\theta = 45^o$.

6. Assuming that the pumpkin which won the 2006 Punkin Chunkin competition was launched at 45^o, calculate:

   (i) the speed of the pumpkin at launch (i.e., v_0),

   (ii) the maximum height reached by the pumpkin during its flight, and

   (iii) the duration of its flight.

7. Suppose that the object of the competition was to maximize the height reached by the pumpkin, rather than its horizontal range. Assuming Second Amendment always launches its pumpkin with the same speed regardless of angle, what height would it have achieved?

Numerical Answers

6. (i) 108 m s^{-1}, (ii) 295 m, (iii) 15.5 s

7. 590 m