Mechanics Classwork II. Answers

1. $\frac{1}{2} m_{Xe} v_e^2 = QV$

m_{Xe} = mass of xenon ion $= 2.18 \times 10^{-25}$ kg
$Q = 1.60 \times 10^{-19}$ C
$V = 10^3$ V

$\therefore v_e = \left(\frac{2 \times 1.6 \times 10^{-19} \times 10^3}{2.18 \times 10^{-25}} \right)^{\frac{1}{2}} = 3.83 \times 10^4 \text{ ms}^{-1}$

$v_e/c = 1.28 \times 10^{-4} \ll 1$

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2. $m\,dv = -v_e\,dm$

Rearrange & integrate assuming v_e is const

$$\int_{v_{init}}^{v_{final}} dv = -v_e \int_{M_{init}}^{M_{final}} \frac{dm}{m}$$

$LHS = \Delta v$, $RHS = -v_e \ln(M_{final}/M_{init}) = v_e \ln(M_{init}/M_{final})$

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3. Over the lifetime of the spacecraft:

$M_{init} = M + m_{prop}$, $M_{final} = M$

$$\Delta v_{tot} = v_e \ln\left(\frac{M + m_{prop}}{M}\right)$$

$$\therefore \ln\left(1 + \frac{m_{prop}}{M}\right) = \frac{\Delta v_{tot}}{v_e}$$

$$\therefore 1 + \frac{m_{prop}}{M} = e^{\Delta v_{tot}/v_e} \quad \therefore m_{prop} = M\left(e^{\Delta v_{tot}/v_e} - 1\right)$$

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4. $M = 2 \times 10^3$ kg

$\Delta v_{tot} = 2.0 \times \frac{52}{2} \times 15 = 780 \text{ ms}^{-1}$

(i) Conventional rocket: $v_e = 2.0 \times 10^3 \text{ ms}^{-1}$

$m_{prop} = 2 \times 10^3 \left(e^{780/2.0 \times 10^3} - 1\right) = 956$ kg.

(ii) Ion thruster: $v_e = 3.83 \times 10^4 \text{ ms}^{-1}$

$m_{prop} = 2 \times 10^3 \left(e^{780/3.83 \times 10^4} - 1\right) = 41$ kg

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5. Satellite fitted with ion thruster need 913 kg less propellant on board

$\Rightarrow$ save $\$2.5 \times 10^4 \times 913 \simeq \$23$ million

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