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First Year Physics 1.11.07

Classwork II
Ion thrusters

Information needed for this Classwork

Basic equation for rocket motion: $mdv = -v_e dm$
Mass of Xenon ion = = 2.18×10^{-25} kg.
Speed of light = 3.0×10^8 m s^{-1}.

Rockets work by expelling stuff at a high velocity. The velocity with which the stuff is expelled relative to the rocket is called the exhaust velocity, denoted v_e. In this classwork we refer to the stuff as "propellant".

In an ion thruster the propellant is a stream of positively charged ions from a plasma. The ions acquire their kinetic energy by being accelerated electrically. The propellant in a conventional, chemical rocket, on the other hand, acquires its kinetic energy by being burnt. In this case the propellant is often termed the "fuel", because it is also the energy source, but this term is not appropriate for an ion thruster with its electrical energy source.

The exhaust velocity in a conventional rocket is limited to a few km s^{-1}. Considerably higher exhaust velocities are possible with an ion thruster. For many applications this is very significant, and ion thrusters are now being used more and more in space. This classwork explores the reason for their increasing popularity.

1. Many ion thrusters use the inert gas Xenon as a propellant. It is ioized to form a plasma in which the ions are singly charged (i.e., the Xenon ions have a charge of $+1.60 \times 10^{-19}$ C). Consider such a device in which the ions are accelerated with a potential difference of 1 kV. Using the fact that a charge Q which is accelerated through a potential difference of V acquires a kinetic energy $K = QV$, and assuming that the motion is non-relativistic (so that the equations of classical mechanics can be used) calculate the exhaust velocity of the Xenon from the thruster. Check that $v_e << c$, thus justifying the neglect of relativistic effects.

2. Manoeuvres in space are achieved by changing the spacecrafts velocity, and are characterized by $\Delta v = v_{final} - v_{init}$. Assuming that v_e is constant, show that

$$\Delta v = v_e \ln\left(\frac{m_{init}}{m_{final}}\right)$$

   where m_{init} is the mass of the spacecraft + propellant before the manoeuvre and m_{final} is the mass after the manoeuvre.

3. We can write $m = M + m_{prop}$, where M is the 'dry mass' (the payload and the spacecraft structure), and m_{prop} is the mass of the propellant. Over the lifetime of the spacecraft all the propellant will be used up. Show that the mass of propellant required to achieve a total Δv of Δv_{tot} over the lifetime of the spacecraft is:

$$m_{prop} = M\left(e^{\Delta v_{tot}/v_e} - 1\right) \ .$$

4. Satellites tend to drift out of their correct orbits and they are fitted with rockets to make regular orbit adjustments to compensate for this. Consider a satellite with a dry mass of 2 tonnes ($= 2 \times 10^3$ kg) which requires an orbit correcting manoeuvre with a Δv of 2.0 m s^{-1} once every 2 weeks. The satellite has a lifetime of 15 years. Calculate the mass of propellant required if:

   (i) a conventional chemical rocket with $v_e = 2.0 \times 10^3$ m s^{-1} is used, or,

   (ii) the ion thruster discussed in Q. 1 is used.

5. Launching a satellite costs approximately \$ 25,000 per kg. Estimate (to the nearest million dollars) how much you could save on the launch if your satellite was fitted with an ion thruster rather than a conventional rocket.

Numerical Answers

1. 3.83×10^4 m s^{-1}

4. (i) 954 kg, (ii) 41 kg

5. \$ 23 million