Mechanics | M. Coppins
First Year Physics | 25.10.07

Classwork I
Gravity, weight and tidal forces

Information needed for this Classwork

The magnitude of the gravitational force between two masses, m_1 and m_2, distance r apart, is $F = \dfrac{Gm_1m_2}{r^2}$

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Acceleration due to gravity at the Earth's surface = 9.81 m s^{-2}

Mass of the Earth = 5.98×10^{24} kg

Mass of the Moon = 7.36×10^{22} kg

Mass of the Sun = 1.99×10^{30} kg

Radius of the Earth = 6.37×10^{6} m

Radius of the Moon = 1.74×10^{6} m

This classwork consists of miscellaneous questions about gravity. You can assume that the gravitational force on a point particle exerted by a spherical object is the same as that exerted by a point particle located at the sphere's centre with the same mass as the sphere (this will be proved in Lecture 5.1).

1. Calculate the acceleration due to gravity

   (i) at the surface of the Moon, and

   (ii) 10^9 m from the centre of the Sun.

2. Calculate the weight of a 10kg mass

   (i) at the surface of the Earth,

   (ii) at the surface of the Moon, and

   (iii) 10^9 m from the centre of the Sun.

3. In the preamble to the classwork, above, you were told to assume that the gravitational force on a point particle exerted by a spherical object is the same as that exerted by a point particle located at the sphere's centre with the same mass as the sphere. Does this mean that the gravitational force between *two* spherical objects is the same as that between two point particles with the same masses as the spheres and separated by the distance between their centres? [Hint: you don't need to do any maths to answer this question.]

Continued overleaf

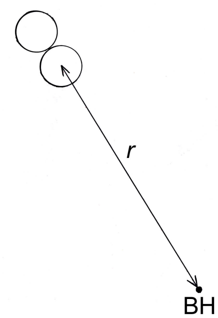
4. A 1 kg Christmas pudding falls from a shelf in Waitrose. While it falls,

   (i) what is the acceleration of the pudding due to the Earth's gravity,

   (ii) what is the acceleration of the Earth due to the pudding's gravity.

   (iii) A constant acceleration (magnitude a) will cause an object, initially at rest, to move a distance $x = \frac{1}{2}at^2$ in time t (see Lecture 2.1). If the acceleration found in part (ii) continued indefinitely, how long would it be before it had caused the Earth to move 10^{-10} m. (approximately the width of an atom).

5. A composite object consisting of 2 spheres (each of radius a) joined together, falls under gravity towards a supermassive black hole of mass M. It falls in such a way that the axis joining the centres of the spheres points towards the black hole. The centre of the nearer sphere is distance r from the black hole (see diagram). Assuming that $a << r$, and using a binomial expansion, show that the acceleration due to gravity at the position of the centre of the other (further) sphere is $g_r - \dfrac{4GMa}{r^3}$, where g_r is the acceleration at the centre of the nearer sphere.



6. The fact that the nearer sphere experiences a stronger gravitational force means that there is an effective force trying to break the bond joining the spheres. Such forces are called *tidal forces*, and they will be discussed in more detail in Lecture 5.3. For now, just suppose you were unfortunate enough to be falling into a supermassive black hole of mass 10^{10} solar masses. As you fall the tidal forces will tear you apart. Very close to the black hole the tidal forces will be strong enough to separate adjacent molecules. Estimate the distance of your fragmentary remains from the black hole when this occurs. Assume you are composed entirely of water, and that water molecules can be approximated by spheres of radius 2×10^{-10} m, held together by a force of approximately 10^{-11} N. The mass of a water molecule is approximately 3×10^{-26} kg.

Numerical Answers

1. (i) 1.62 m s^{-2}, (ii) 132 m s^{-2}

2. (i) 98.1 N, (ii) 16.2 N,(iii) 1.32×10^3 N

4. (i) 9.81 m s^{-2}, (ii) 1.64×10^{-24} m s^{-2}, (iii) 1.01×10^7 s (about $4\frac{1}{2}$ months)

6 147 m (in fact, classical mechanics breaks down this close to a black hole, but you can get some idea of just how extreme these things are.)