Complete this cover very clearly and, in your own interest, follow carefully the instructions printed below.

IMPORTANT: Enter here the name and type of Electronic Calculator used – if any. *(instruction 7 below).*

027

Candidate's Number ....

College ....

Examination *(Insert official title of the Examination for which you have entered, as it appears on the question paper).* ....

Title of {* Morning / * Afternoon} Paper MECHANICS + RELATIVITY
*(*Delete the word which does not apply)*

Section *(if any)* ....

Date 2004

UNIVERSITY OF LONDON

INSTRUCTIONS TO CANDIDATES

1. **In no circumstances may you remove answer books, used or unused, from the Examination Room.**
2. Write on both sides of the paper.
3. Begin each answer on a fresh page.
4. Write the number of the question at the top of *each* page.
5. Do all rough work in the Answer Book and cross it through.
6. If you use supplementary books, tie them to the end of this book inside the cover. Do not write answers in a supplementary book until you have used all the pages in this book.
7. In any examination where the use of Electronic Calculators is permitted, the box at the top right-hand corner of this cover must be completed.
8. In the box below, enter the numbers and sub-sections of the questions in the order in which you have attempted them.

Qu 3

You are warned that the Examiners attach great importance to legibility, accuracy and clarity of expression.

For Examiner's use only

Question No.	Mark
3	31
Total	31

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Question 3

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③ i) $m\frac{dv_y}{dt} = -mg - bv_y$ ✓ 2

↑ +ve y.

⇒ At terminal velocity there is no net force on body.

⇒ $0 = -mg - bv_y$ ✓ 1

$v_y = -\frac{mg}{b}$ ✓ 1

4/4

ii) $\frac{dv_y}{dt} = -g - \frac{b}{m}v_y$ ✓ 1

$\int_{v_{0y}}^{v_y(t)} \frac{1}{g + \frac{b}{m}v_y} dv_y = -\int_0^t dt + c$ = → 1 2

⇒ $\frac{m}{b} \ln \frac{\left(g + \frac{b}{m}v_y(t)\right)}{\left(g + \frac{b}{m}u\right)} = -t$ ✓

Limits 2

⇒ $\frac{g + \frac{b}{m}v_y(t)}{g + \frac{b}{m}u} = e^{-\frac{b}{m}t}$ ✓

⇒ $g + \frac{b}{m}v_y(t) = \left(g + \frac{b}{m}u\right)e^{-\frac{b}{m}t}$

⇒ $v_y(t) = \frac{m}{b}\left(g + \frac{b}{m}u\right)e^{-\frac{b}{m}t} - \frac{mg}{b}$ 3

$= \left(\frac{mg}{b} + u\right)e^{-\frac{b}{m}t} - \frac{mg}{b}$ ✓

9/9

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Question.................

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iii) ~~And~~ As $t \to \infty$, $e^{-\frac{bt}{m}} \to 0$ ✓ 1

$\Rightarrow V_y(t) = \frac{-mg}{b}$ ✓ 1

iv) At maximum height $V_y(t) = 0$ ✓ 2

$\Rightarrow 0 = \left(\frac{mg}{b} + u\right) e^{-\frac{b}{m}t} - \frac{mg}{b}$

$\frac{mg}{b\left(\frac{mg}{b} + u\right)} = e^{-\frac{b}{m}t}$ ✓ (t_{max} −1) 3

$\Rightarrow \ln\left(\frac{mg}{mg + bu}\right) = -\frac{b}{m}t$

$\Rightarrow t_{max} = -\frac{m}{b}\left(\ln\left(\frac{mg}{mg+bu}\right)\right)$

$t_{max} = \frac{-m}{b} \ln\left(\frac{1}{\left(1 + \frac{bu}{mg}\right)}\right)$ ✓ 3

8/9

v) $e^x = 1 + x + \frac{x^2}{2} \ldots$

$e^{-\frac{b}{m}t} = 1 - \frac{b}{m}t + \frac{b^2t^2}{2m^2} \ldots$ ✓ 1

In the limit $m \gg bt_{max}$ $\frac{b^2t^2}{2m^2} \to 0$ 1

$\Rightarrow 1 - \frac{b}{m}t_{max} = \frac{1}{\left(1 + \frac{bu}{mg}\right)}$ ✓

$$\Rightarrow 1 - \frac{1}{\left(1 + \frac{bu}{mg}\right)} = \frac{b}{m} t_{max}$$

1

$$\Rightarrow 1 + \frac{bu}{mg} - 1 = \frac{b}{m} t_{max} + \frac{bu}{mg}\frac{b}{m} t_{max}$$

$$\Rightarrow \frac{u}{g} = t_{max}\left(1 + \frac{b}{m}\right)$$

If $b t_{max} << m$ then ✓

$t_{max} \approx \frac{u}{g}$ in this limit. ✓

2

• Expected result for neglection of air resistance.

$$u_y^2 = -2g(y_{max}) \Rightarrow y_{max} = \frac{u_y^2}{2g}$$

$v^2 = u^2 + 2as$

$$\Rightarrow y_{max} = \frac{u_y^2}{2g} = u_y t - \frac{1}{2} g t^2$$ ✓

$$t_{max} = \frac{u_y}{g} \pm \frac{\sqrt{u_y^2 - u_y^2}}{g} = \frac{u_y}{g}$$ ✓

3

which is what we have shown.

$v = u + at$ easier

31