PROBLEM SHEET 8

1

$f(x)$	αx^n	$(\alpha x+\beta)^n$	$\frac{1}{x}$	$\frac{1}{\alpha x+\beta}$
$\int f(x)dx$	$\frac{\alpha x^{n+1}}{n+1}+C$	$\frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)}+C$	$\ln x + C$	$\frac{1}{\alpha}\ln(\alpha x+\beta)$ C
$f(x)$	$e^{\gamma x}$	$\cos x$	$\sin x$	
$\int f(x)dx$	$e^{\gamma x}/\gamma + C$	$\sin x + C$	$-\cos x + C$	

2. (a) $x^6/6 + C$ (b) $-x^{-4}/4 + C$

(c) $u = 4x+1$, $du = 4dx \therefore \int = \int 2u^{-1/3}du = 3u^{2/3}+C = 3(4x+1)^{2/3}+C$

(d) $u = 2x^3+4$, $du = 6x^2dx \therefore \int = \int u^{-2}du = -u^{-1}+C = -(2x^3+4)^{-1}+C$

(e) $u = \omega t$, $du = \omega dt \therefore \int = \int \cos u \frac{1}{\omega} du = \frac{1}{\omega}\sin u + C = \frac{1}{\omega}\sin(\omega t)+C$

3. (a) $v = x$, $\frac{du}{dx} = e^{-x} \to \frac{dv}{dx} = 1$, $u = -e^{-x} \therefore \int = -xe^{-x} - \int(-e^{-x})dx$
$= -xe^{-x} - e^{-x} + C$

(b) $v = x$, $\frac{du}{dx} = \cos x \to \frac{dv}{dx} = 1$, $u = \sin x \therefore \int = x\sin x - \int \sin x\, dx$
$= x\sin x + \cos x + C$

(c) $v = \ln x$, $\frac{du}{dx} = x^2 \to \frac{dv}{dx} = \frac{1}{x}$, $u = \frac{x^3}{3}$

$\therefore \int = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

4 (a) $v = x^2$, $\frac{du}{dx} = \cos x \to \frac{dv}{dx} = 2x$, $u = \sin x$. $\int = x^2\sin x - \int \sin x\, 2x\,dx$.

do integral on RHS by parts: $\int x\sin x\,dx = -x\cos x - \int(-\cos x)dx$
$= -x\cos x + \sin x$

$\therefore$ original integral $= x^2\sin x + 2x\cos x - 2\sin x + C$

(b) $v = \ln x$, $\frac{du}{dx} = 1 \to \frac{dv}{dx} = \frac{1}{x}$, $u = x$ $\therefore \int = x\ln x - \int x\frac{1}{x}dx = x\ln x - x + C$

(c) $I = \int e^x\sin x\,dx$, $v = \sin x$, $\frac{du}{dx} = e^x \to \frac{dv}{dx} = \cos x$, $u = e^x$.

$\therefore I = e^x\sin x - \int e^x\cos x\,dx$ — $e^x\cos x - \int e^x(-\sin x)dx$ (integrating by parts again)

$\therefore I = e^x\sin x - \{e^x\cos x + \int e^x\sin x\,dx\} = e^x\sin x - e^x\cos x - I$ ($\int e^x\sin x\,dx = I$)

$\therefore 2I = e^x(\sin x - \cos x)$ $\therefore I = \frac{1}{2}e^x(\sin x - \cos x)$.

5 (a) See Q2(a) $\to \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$

(b) See Q2(d) $\to \left[-\frac{1}{(2x^3+4)}\right]_{-1}^1 = -\left\{\frac{1}{2+4} - \frac{1}{2(-1)+4}\right\} = \frac{1}{3}$

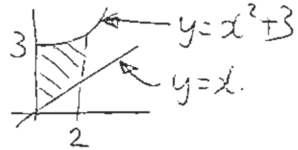
(c) See Q2(e) $\to \left[\frac{1}{2}\sin(2t)\right]_0^{\pi/4} = \frac{1}{2}\left\{\sin\frac{\pi}{2} - \sin 0\right\} = \frac{1}{2}$

(d) See Q3(a) $\to \left[-xe^{-x} - e^{-x}\right]_0^2 = -\{2e^{-2} - e^{-2} - 0.e^0 - e^0\} = 1 - 3e^{-2}$
$\simeq 0.594$

(e) See Q4(b) $\to [x\ln x - x]_1^2 = 2\ln 2 - 2 - \ln 1 + 1 = 2\ln 2 - 1 \simeq 0.386$

6 (a) area $= \int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{1}{3}(8-1) = 7/3$

(b) area $= \int_2^5 2e^x dx = [2e^x]_2^5 = 2e^5 - 2e^2 \simeq 282.05$

(c) 

area under $y = x^2+3$ is $\int_0^2 (x^2+3)dx$
$= \left[\frac{x^3}{3} + 3x\right]_0^2 = \frac{8}{3} + 6 = \frac{26}{3}$

area under $y = x$ is $\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 = 2$ $\therefore$ area between curves is $\frac{26}{3} - 2 = 20/3$

7 Put $\theta = \phi = x \to \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 \to \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\therefore \int_0^{2\pi}\cos^2 x\,dx = \int_0^{2\pi}\frac{1}{2}dx + \int_0^{2\pi}\frac{1}{2}\cos 2x\,dx = \left[\frac{x}{2}\right]_0^{2\pi} + \left[\frac{1}{2}\cdot\frac{1}{2}\sin 2x\right]_0^{2\pi} = \pi$

8 (a) Strip 1, area $\simeq \frac{d}{2}(y_0+y_1)$, Strip 2, area $\simeq \frac{d}{2}(y_1+y_2)$, Strip 3, area $\simeq \frac{d}{2}(y_2+y_3)$

$\therefore$ total area $\simeq \frac{d}{2}\{y_0 + 2y_1 + 2y_2 + y_3\}$

(b) $A \simeq \frac{d}{2}\{y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + y_5\}$

(c) $y_0 = 1$, $y_1 = 1.44$, $y_2 = 1.96$, $y_3 = 2.56$, $y_4 = 3.24$, $y_5 = 4.0$, $d = 0.2$

$\Rightarrow A \simeq 2.34$ [actually area $= \frac{7}{3} \simeq 2.333$]