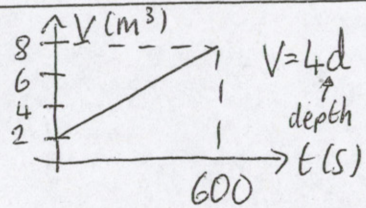
PROBLEM SHEET 6

1. $SP-S = 14.7/\cos 48^o = 22.0$ km
$H-S = 14.7 \tan 48^o = 16.3$ km
[The SP-H distance, 14.7 km, can be found from measurements of angles at 2 other trig pt's & in fact, all the distances can be found by measuring one distance on the ground (the 'base line') & then just measuring angles. This is how the original Ordnance Survey trigonometric survey of the whole country was done. It used a base line measured in 1784 in Hounslow, then an entirely rural area.]

2(a) 

(b) Slope of this graph is rate of flow $= \frac{6}{600} = 0.01\ m^3/s$

(c) $V = 2.0 + 0.01t$

3 v = usual speed (km/hr), t = usual time (hours)
Usually: $v = \frac{80}{t}$, On this occasion: $v - 20 = \frac{80}{t+0.5}$

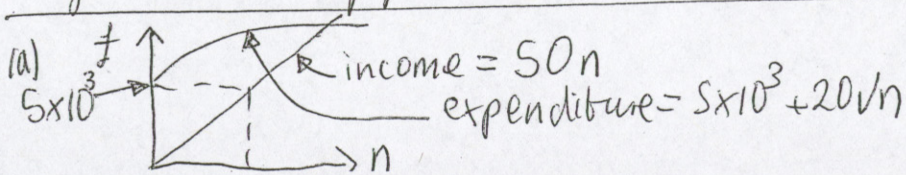
Subs for v: $\frac{80}{t} - 20 = \frac{80}{t+0.5} \rightarrow \frac{80(t+0.5) - 20t(t+0.5) - 80t}{t(t+0.5)} = 0$

$\times \frac{t(t+0.5)}{10} \rightarrow 8(t+0.5) - 2t(t+0.5) - 8t = 0$

$\rightarrow 8t + 4 - 2t^2 - t - 8t = 0 \rightarrow 2t^2 + t - 4 = 0$

$\therefore t = \frac{-1 \pm (1 - 4.2.(-4))^{1/2}}{4} = \frac{-1 \pm 33^{1/2}}{4} = 1.19$ or -1.69 hrs

Reject 2nd soln as unphysical → usual time = 1.19 hrs ≃ 1 hr 11 mins.

4 (a) 

(b) Min no of orders/week when income = expenditure

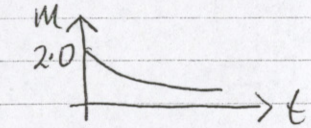
i.e. $50n = 5\times10^3 + 20\sqrt{n} \rightarrow 5n - 2\sqrt{n} - 500 = 0$
Define $x = \sqrt{n}$ → quad eq: $5x^2 - 2x - 500 = 0$

$\rightarrow x = \frac{2 \pm (4 - 4.5.(-500))^{1/2}}{10} = \frac{2 \pm (10004)^{1/2}}{10} = 10.2$ or -9.8

Reject -ve soln ($\sqrt{n}$ must be +ve) → $n_{min} = 10.2^2 \simeq 104$

5 (a) s^{-1} (remember, radians are dimensionless)
(b) $I(t=0) = I_0 \sin(0) = 0$
(c) when $\omega t = \pi$ then $\sin(\omega t) = 0$ ∴ $I = I(t=0) = 0$ when $t = \pi/\omega$
(d) next time is $t = 2\pi/\omega$
(e) In general $I = 0$ when $t = n\pi/\omega$ (n = integer)

6 (a) $P(t=0) = 10^3$
(b) $P = 10^6 \rightarrow 10^6 = 10^3 e^{0.01t} \rightarrow 0.01t = \ln\left(\frac{10^6}{10^3}\right)$
$\rightarrow t = 691$ years
(c) $P = 2\times10^6 \rightarrow 0.01t = \ln\left(\frac{2\times10^6}{10^3}\right) \rightarrow t = 760$ years
i.e. it only takes another 69 years to reach 2 million.

7 (a) $M(t=0) = 2.0$ kg. (b) 

(c) $M(t=1200) = 2.0e^{-1.2} = 0.60$ kg
(d) when $M = 1.0 \rightarrow 1.0 = 2.0e^{-0.001t}$
$\rightarrow -0.001t = \ln 0.5 \rightarrow t = 693$ s ~ 11.5 mins

8 (a) $\frac{1}{\gamma}\ln 2 = 1.5$ years $\rightarrow \gamma = 0.462$ years^{-1}

(b) 1965 → 2000 = 35 years. Assume $n = n_0 e^{\gamma t}$

$\rightarrow 4.2\times10^7 = 64 e^{\gamma.35} \rightarrow 35\gamma = \ln\left(\frac{4.2\times10^7}{64}\right) \rightarrow \gamma = 0.383$ years^{-1}
→ doubling period $= \frac{1}{\gamma}\ln 2 \simeq 1.8$ years ≃ 22 months