PROBLEM SHEET 5

1. Sum of roots $= \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)+\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = \frac{-2b}{2a} = \frac{-b}{a}$

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2. (a) $b^2-4ac = 100-4.2.12 = 4 \rightarrow$ 2 distinct real roots

$x = \frac{10\pm\sqrt{4}}{4} = \frac{12}{4}$ or $\frac{8}{4} = 3$ or 2. Sum $= 5 = \frac{-b}{a}$

(b) Eq is $6x^2+x-1=0$ ∴ $b^2-4ac = 1-4.6.(-1) = 25 \rightarrow$ 2 distinct real roots

$x = \frac{-1\pm\sqrt{25}}{12} = \frac{4}{12}$ or $-\frac{6}{12} = \frac{1}{3}$ or $\frac{-1}{2}$. Sum $= \frac{-1}{6} = \frac{-b}{a}$

(c) $b^2-4ac = 64-4.1.16 = 0 \rightarrow$ repeated root: $x = \frac{-b}{2a} = -4$. Sum $= -8 = \frac{-b}{a}$

(d) Eq is $5x^2-2x=0$ i.e. $a=5, b=-2, c=0$

factorize $x(5x-2)=0$, $x=0$ or $\frac{2}{5}$. Sum $= \frac{2}{5} = \frac{-b}{a}$

(e) $a=1, b=0, c=-25$, roots are $x=+5$ or -5. Sum $= 0 = -b/a$.

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3 (a) $= 2g(x) = 2(1+3x)$ (b) $= 1+3h(x) = 1+\frac{3}{x} = \frac{x+3}{x}$

(c) $= \frac{1}{g(x)} = \frac{1}{1+3x}$ (d) $g(h(x)) = \frac{x+3}{x}$, $f(g(h(x))) = \frac{2(x+3)}{x}$

(e) $g(f(x)) = 1+3f(x) = 1+6x$, $h(g(f(x))) = \frac{1}{1+6x}$

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4 (a) $y=u^4$, $u=x+2$ (b) $y=u^{-2}$, $u=3x+1$

(c) $y=2^u$, $u=x-1$ (d) $y=\sin u$, $u=x^2$

(e) $y=1-2u$, $u=\log_{10}x$ [or: $y=\log_{10}u$, $u=10/x^2$]

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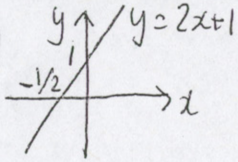
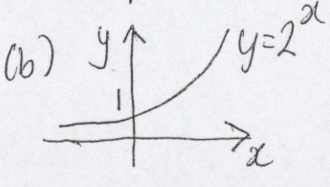
5 (a)



One-to-one → has inverse

$x=2y+1 \rightarrow y=\frac{1}{2}(x-1)$

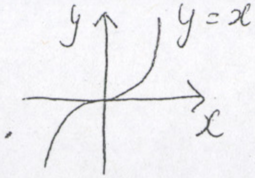
(b)



One-to-one → has inverse

$x=2^y \rightarrow y=\log_2 x$

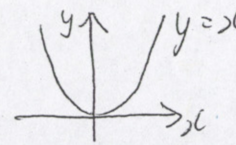
(c)



One-to-one → has inverse

$x=y^3 \rightarrow y=x^{1/3}$

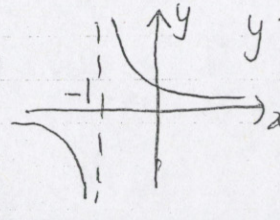
(d)



Not one-to-one → no inverse

[does have one for $x>0$]

(e)



$y=\frac{1}{x+1}$ [$\frac{1}{x}$ shifted by 1 to left].

One-to-one → has inverse

$x=\frac{1}{y+1} \rightarrow y=\frac{1}{x}-1 = \frac{1-x}{x}$.

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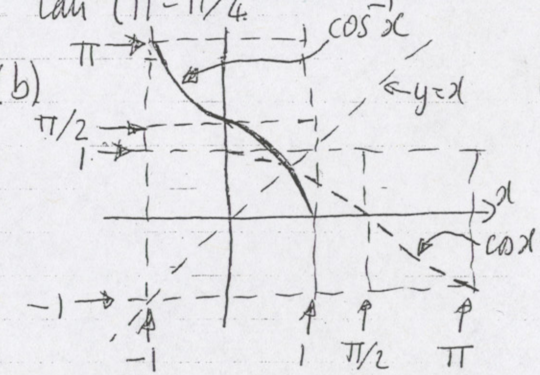
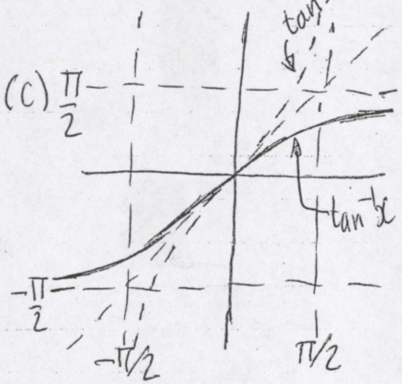
6 (a) $\cos^{-1}(0)=\pi/2$, $\cos^{-1}(x)$ not defined for $x>1$, $\tan^{-1}(0)=0$,

$\tan^{-1}(1)=\pi/4$

(b)



(c)



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7 (a) x axis (b) y axis (c) −ve x axis.

(d) $y=x$ (e) $y=\pi/2$.

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8 (a) TRUE. $x=2$ in both cases

(b) FALSE: $f(f(x)) = 1/f(x) = x$

(c) TRUE

(d) TRUE: max. value of $\sin x$ is 1, so $\sin^{-1}x$ not defined for $x>1$

(e) FALSE: it is an odd fn. (see Q.6)