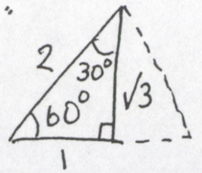
# PROBLEM SHEET 4

1. 


$\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\tan 60^\circ = \sqrt{3}$

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2 (a) $45^\circ = \pi/4$ rad, $-10^\circ = -0.174$ rad, $720^\circ = 4\pi$ rad
(b) $0.1$ rad $= 5.73^\circ$, $6\pi$ rad $= 1080^\circ$, $-7\pi/2$ rad $= -630^\circ$

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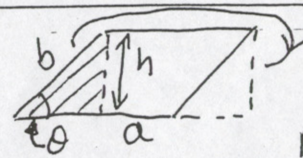
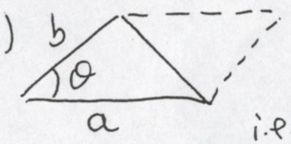
3 (a) 


Moving triangle → rectangle equal in area i.e. area of ∥gram $= ah$
But $h = b\sin\theta$ ∴ area $= ab\sin\theta$

(b) 


Area of triangle $= \frac{1}{2} \times$ area of ∥gram
$= \frac{1}{2} ab\sin\theta$.
i.e. $\frac{1}{2} \times$ product of sides $\times$ sin of included angle

(c) Area of whole circle $= \pi r^2$, fraction occupied by sector $= \theta/2\pi$
($\theta$ in radians) ∴ area of sector $= \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2}\theta r^2$

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4 (a) $\operatorname{cosec} 45^\circ = \sqrt{2}$, $\sec 30^\circ = 2/\sqrt{3}$, $\cot 60^\circ = 1/\sqrt{3}$.
(b) $\sin^2\theta + \cos^2\theta = 1$. Divide by $\cos^2\theta \rightarrow \tan^2\theta + 1 = \sec^2\theta$.

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5 (a) Area of $\Delta = \frac{1}{2} \times$ product of 2 sides $\times$ sin of included angle (Q3, b)
Area ABD $= \frac{1}{2} hc\sin\theta$, but $h = a\cos\phi$ ∴ area $= \frac{1}{2} ac\sin\theta\cos\phi$
Area DBC $= \frac{1}{2} ha\sin\phi$, $h = c\cos\theta$ ∴ area $= \frac{1}{2} ac\cos\theta\sin\phi$
Area ABC $= \frac{1}{2} ac\sin(\theta+\phi)$
But area ABC = area ABD + area DBC
Cancel $\frac{1}{2}ac \Rightarrow \sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

(b) $\underbrace{\sin\left(\frac{\pi}{2} - \theta - \phi\right)}_{\cos(\theta+\phi)} = \underbrace{\sin\left(\frac{\pi}{2} - \theta\right)}_{\cos\theta} \underbrace{\cos(-\phi)}_{\cos\phi} + \underbrace{\cos\left(\frac{\pi}{2} - \theta\right)}_{\sin\theta} \underbrace{\sin(-\phi)}_{-\sin\phi}$

(c) ∴ $\cos 2\phi = \cos^2\phi - \sin^2\phi = 1 - 2\sin^2\phi$. Let $\phi = \frac{\theta}{2}$
$\rightarrow \cos\theta = 1 - 2\sin^2\theta/2$

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6 (a) area of triangle $= \frac{1}{2} r^2 \sin\theta$ (Q3, b)
area of sector $= \frac{1}{2} r^2\theta$ (Q3, c) [$\theta$ IN RADIANS]

∴ $\frac{\text{area of triangle}}{\text{area of sector}} = \frac{\sin\theta}{\theta}$

(b) As $\theta$ is made smaller the 2 areas become closer i.e. for very small $\theta$ we have $\sin\theta/\theta \simeq 1 \rightarrow \sin\theta \simeq \theta$.

(c) ∴ for small angles $\cos\theta \simeq 1 - 2(\theta/2)^2 = 1 - \theta^2/2$

(d) TRUE. REMEMBER, THE SMALL ANGLE APPROXIMATIONS ARE ONLY VALID IF $\theta$ IS IN RADIANS
To 6 dec. places:
$1^\circ \rightarrow \theta = 0.017453$, $\sin\theta = 0.017452$
$1 - \theta^2/2 = 0.999848$, $\cos\theta = 0.999848$

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7 (a) $\sqrt{8}$, (b) 5, (c) $\sqrt{5}$, (d) $(4-2)/(3-2) = 2$, (e) $\tan\theta = \Delta y/\Delta x$ = grad = 2, (f) $y = 2x - 2$

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8 (a) $d = \{(x-a)^2 + (y-b)^2\}^{1/2} = \{x^2 - 2ax + a^2 + y^2 - 2by + b^2\}^{1/2}$
On circle values of $x$ & $y$ must satisfy
$\{x^2 + y^2 - 2ax - 2by + a^2 + b^2\}^{1/2} = r$
$\rightarrow x^2 + y^2 - 2ax - 2by = r^2 - a^2 - b^2$ i.e. $\alpha = -2a$, $\beta = -2b$, $\gamma = r^2 - a^2 - b^2$

(b) (i) $\alpha = -2 \rightarrow a = 1$, $\beta = 0 \rightarrow b = 0$, $\gamma = 3 \rightarrow r^2 = 3 + 1 \rightarrow r = 2$
i.e. $r = 2$, centre at $(1, 0)$
(ii) $\alpha = 4 \rightarrow a = -2$, $\beta = -8 \rightarrow b = 4$, $\gamma = -11 \rightarrow r^2 = -11 + 4 + 16 = 9 \rightarrow r = 3$
i.e. $r = 3$, centre $(-2, 4)$
(iii) $\alpha = 2 \rightarrow a = -1$, $\beta = 4 \rightarrow b = -2$, $\gamma = -9 \rightarrow r^2 = -9 + 1 + 4 = -4$
→ no solution. There are no values of $x$ & $y$ satisfying this eq.