PROBLEM SHEET 3

1. (a) $f(0) = 4$, (b) $f(2x) = 2(2x+4) = 4x+4$,
   (c) $f(-2x) = 2(-2x)+4 = -4x+4$
   (d) $f(1/x) = 2(1/x)+4 = 2/x + 4$
   (e) $f(3a-2) = 2(3a-2)+4 = 6a$

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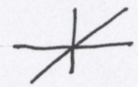
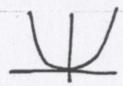
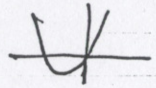
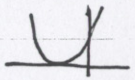
2. (a) ODD  (b) EVEN 
   (c) EVEN  (d) NEITHER 
   (e) NEITHER 

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3. All have form $y = ax+2$, where a = gradient.
   (a) $y = x+2$, (b) $y = 4x+2$, (c) $y = -2x+2$,
   (d) $y = 2$, (e) $x = 0$ i.e. the y axis.

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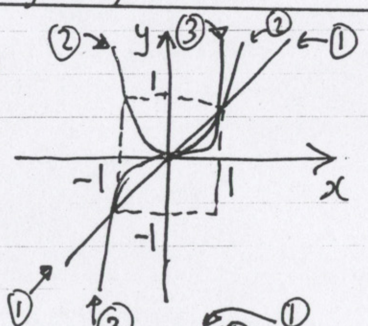
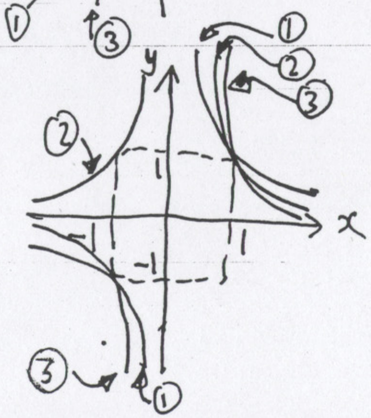
4 (a)



① : $y = x$
② : $y = x^2$
③ : $y = x^3$

(b)



① $y = x^{-1}$
② $y = x^{-2}$
③ $y = x^{-3}$

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5. Remember $\alpha = \frac{b}{2a}$, $\beta = \frac{4ac-b^2}{4a^2}$, $\gamma = a$ (PS2)
   (i) y has extreme value of $\frac{4ac-b^2}{4a}$ $(=\gamma\beta)$.
   (ii) this extreme is minimum if $a>0$, max. if $a<0$
   (iii) curve crosses x axis if $\frac{4ac-b^2}{4a^2} < 0$

   Since $a \neq 0$ (otherwise it's not a quadratic fn.) & $4a^2 > 0$ the condition for curve to cross x axis becomes
   $4ac-b^2 < 0$ or $b^2 > 4ac$.

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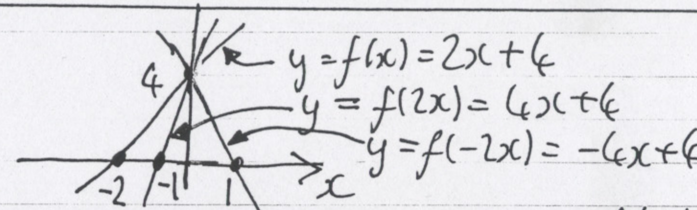
6 (a)



(b) $y = f(\alpha x)$: curve $y = f(x)$ is compressed by factor α in x dim. (reflected in y axis for $\alpha < 0$)
(c) $f(x) = x^2$ is an even fn (see Q2, above). So, e.g. transformations $y = f(2x)$ & $y = f(-2x)$ would look identical. To illustrate effect of sign of α on transformation need a fn which is not even.

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7. (a) $Y = \log_{10} y = \log_{10} ax^b = \log_{10} a + \log_{10} x^b = A + bX$, where $A = \log_{10} a$
   ∴ graph of Y against X is straight line with grad b & Y axis intercept of $A = \log_{10} a$.
   (b) $Y = -1 - 0.5X \Rightarrow y = ax^b$ with $\log_{10} a = -1 \Rightarrow a = 0.1$
   & $b = -0.5$, i.e. $f(x) = 0.1x^{-1/2}$
   (c) Eq of str. line $Y = \alpha x + \beta$ ∴ $\log_{10} y = \alpha x + \beta$
   ∴ $y = g(x) = 10^{\alpha x + \beta} = 10^{\alpha x} \times 10^{\beta}$
   i.e. $g(x) = ba^x$ where $b = 10^{\beta}$ & $a = 10^{\alpha}$

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8 (a) TRUE, (b) FALSE: $f(x)$ is zero when arg of fn is $5 \Rightarrow f(x-3)$ is zero when $x-3=5 \Rightarrow x=8$, (c) FALSE: see e.g. Q1(d), above, (d) TRUE, (e) TRUE: both have gradient 2.