PROBLEM SHEET 2 SOLUTIONS.

1. (a) $= 9x^2 + 12x + 4 \therefore$ coeff is 9
   (b) $= (x+2)(x^2+2x+1) = x^3 + 2x^2 + x + 2x^2 + 4x + 2 \therefore$ coeff is 4
   (c) P's triangle $\to (1+x)^{10} = 1 + 10x + \dots$
   $\therefore$ coeff of x^2 in $x(1+x)^{10}$ is 10
   (d) $= x^6 \left(\frac{x-1}{x}\right)^4 = \frac{x^6}{x^4}(x-1)^4 = x^2(x-1)^4 \therefore$ coeff is 1
   (e) $= (ax)^3 + 3(ax)^2 b + 3axb^2 + b^3 \therefore$ coeff is $3a^2b$

2. (a) $= 2(x^2+2x+1) = 2(x+1)^2$
   (b) $= x(a^2x^2 + 2abx + b^2) = x(ax+b)^2$
   (c) $= (4x)^2 - 3^2 = (4x+3)(4x-3)$
   (d) $= (x^2-a^2)^2 = (x-a)^2(x+a)^2$
   (e) $= (x-a)(x-b)$

3. (a) $= \frac{\sqrt{5}+\sqrt{2}}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{\sqrt{5}+\sqrt{2}}{5-2} = \frac{\sqrt{5}+\sqrt{2}}{3}$
   (b) $= \frac{3(2-\sqrt{3})}{(2^2-3)} = 3(2-\sqrt{3})$
   (c) $= \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} = 3 + 2\sqrt{3}\sqrt{2} + 2 = 5 + 2\sqrt{6}$
   (d) $= \frac{(3+5\sqrt{2})(4+2\sqrt{2})}{4^2-(2\sqrt{2})^2} = \frac{12+6\sqrt{2}+20\sqrt{2}+10\sqrt{2}\sqrt{2}}{16-8} = 4 + \frac{13}{4}\sqrt{2}$
   (e) $= \frac{\sqrt{2}(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \frac{3.2+2\sqrt{6}}{18-12} = 1 + \frac{1}{3}\sqrt{6}$

4. $\gamma\{(x+\alpha)^2+\beta\} = \gamma\{x^2+2\alpha x+\alpha^2+\beta\} = \gamma x^2 + 2\gamma\alpha x + \gamma(\alpha^2+\beta)$
   comparing with $ax^2+bx+c \to a=\gamma,\ b=2\gamma\alpha,\ c=\gamma(\alpha^2+\beta)$
   $\therefore \gamma = a,\ \alpha = \frac{b}{2\gamma} = \frac{b}{2a},$
   $\alpha^2+\beta = \frac{c}{\gamma} \Rightarrow \beta = \frac{c}{\gamma} - \alpha^2 = \frac{c}{a} - \frac{b^2}{4a^2} = \frac{4ac-b^2}{4a^2}$

5. $\left(2+\frac{p}{2}\right)^4 = 2^4\left(1+\frac{p}{4}\right)^4 = 2^4\left\{1 + 4\frac{p}{4} + 6\left(\frac{p}{4}\right)^2 + 4\left(\frac{p}{4}\right)^3 + \left(\frac{p}{4}\right)^4\right\}$
   $= 16 + 16p + 6p^2 + p^3 + \frac{p^4}{16}$

6. (a) Equate the 2 expressions & multiply thro' out by $(x+2)(x-3)$
   (b) $x=-2 \Rightarrow -5A = -6+1 \Rightarrow A = 1$
   (c) $x=3 \Rightarrow 5B = 9+1 \Rightarrow B = 2$
   (d) $\frac{x+4}{(x+3)(x-5)} = \frac{A}{(x+3)} + \frac{B}{(x-5)} \Rightarrow A(x-5) + B(x+3) = x+4$
   $x=-3 \Rightarrow -8A = 1 \Rightarrow A = -1/8$
   $x=5 \Rightarrow 8B = 9 \Rightarrow B = 9/8$
   $\Rightarrow -\frac{1}{8(x+3)} + \frac{9}{8(x-5)}$

7. (a) $2 + 2^2 + 2^3 + 2^4 + \dots$
   (b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$
   (c) $2 + 3x + 4x^2 + 5x^3 + \dots$
   (d) $x - 2x^2 + 3x^3 - 4x^4 + \dots$
   (e) $1 - 2x + 3x^2 - 4x^3 + \dots$

8. (a) 1st term $= a$, 2nd term $= ar$, 3rd term $= ar^2$;
   last (i.e. n^{th}) term $= ar^{n-1}$
   (b) $S_n = a + ar + ar^2 + \dots ar^{n-1}$
   $rS_n = ar + ar^2 + ar^3 + \dots ar^n$
   $\therefore S_n - rS_n = a - ar^n \quad \therefore S_n = \frac{a(1-r^n)}{1-r}$
   (c) if $|r|<1$ then $r^n \to 0$ as $n \to \infty \quad \therefore S_\infty = \frac{a}{1-r}$
   (d) $(1-x)^{-1} = \frac{1}{1-x}$ = sum of ∞ GP with $a=1$ & $r=x$
   $= 1 + x + x^2 + x^3 + x^4 + \dots$