PROBLEM SHEET 1 SOLUTIONS

1. (a) Rational
(b) Irrational
(c) Rational (= 5137/1000)
(d) Irrational (irrational + rational = irrational)
(e) Irrational (= $\sqrt{2\times16} = 4\sqrt{2}$)

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2. Try dividing by prime no's in ascending order: 2, 3, 5, 7, 11, ...
(a) 42 is divisible by 2 i.e. $42 = 2\times21$
Now consider 21: not divisible by 2, but is divisible by 3
$21 = 3\times7$. 7 is prime. $\therefore 42 = 2\times3\times7$
(b) 43 is prime
(c) $46 = 2\times22$, $22 = 2\times11$, $\therefore 46 = 2\times2\times11$
(d) $625 = 5\times125$, $125 = 5\times25$, $25 = 5\times5$ $\therefore 625 = 5\times5\times5\times5$
(e) $6! = 6\times5\times4\times3\times2\times1 = 2\times2\times2\times2\times3\times3\times5$

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3. 3:2 means every 5 parts are split as 3 & 2. So the 2 pieces are 3/5 & 2/5 of cake. $\therefore$ larger piece is $\frac{3}{5}\times100 = 60\%$ of cake

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4. (a) $= \frac{5}{35} + \frac{7}{35} = \frac{12}{35}$

(b) $= 2 + \frac{2}{9} = \frac{20}{9}$

(c) $= \frac{1}{(1/6)} - \frac{1}{(9/6 - 5/6)} = 6 - \frac{1}{(4/6)} = 6 - \frac{3}{2} = \frac{9}{2}$

(d) $= \frac{5\sqrt{3}}{3} - 4\sqrt{2} = \frac{5\sqrt{3} - 12\sqrt{2}}{3}$

(e) $= \frac{2a}{2b^2} - \frac{acb}{2b^2} = \frac{a(2-bc)}{2b^2}$

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5. (a) $= \left(\frac{1}{2^{-2}}\right)^3 = (2^2)^3 = 2^6 = 64$

(b) $= 1 + \left(\frac{1}{7}\right)^{-2.0} = 1 + 7^2 = 50$

(c) $= x^{3/2} \times x^{-2/3} = x^{5/6}$ $(\frac{3}{2} - \frac{2}{3} = 5/6)$

(d) $= 3^{2x} \times 3^{4x} = 3^{6x}$

(e) $= (a^5 b^2)^{1/4} \times b^{3/2} \times a^{-2} = a^{5/4} \times a^{-2} \times b^{1/2} \times b^{3/2} = a^{-3/4} b^2$

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6. (a) $b = a^x \rightarrow \log_a b = x$, $c = a^y \rightarrow \log_a c = y$
$bc = a^x a^y = a^{x+y}$
$\therefore \log_a bc = x + y = \log_a b + \log_a c$

(b) $b/c = a^x / a^y = a^{x-y}$
$\therefore \log(b/c) = x - y = \log_a b - \log_a c$

(c) $b^n = (a^x)^n = a^{nx}$
$\therefore \log_a b^n = nx = n \log_a b$

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7. (a) TRUE: $2^4 = 16$
(b) FALSE: $\log_x 5 = y \Rightarrow x^y = 5$
(c) FALSE: $\rightarrow \log_a(4^3 \times 4^{-1}/2) = \log_a 8$
(d) TRUE: $a^0 = 1 \rightarrow \log_a 1 = 0$
(e) TRUE for $a > 1$ (e.g. $\log_{10} 0.1 = -1$) BUT NOT for $a < 1$ (e.g. $\log_{0.1} 10 = -1$).

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8. $c = a^b \rightarrow b = \log_a c$
$\log_{10} c = \log_{10} a^b = b \log_{10} a$ $\therefore b = \frac{\log_{10} c}{\log_{10} a}$
($b = \log_a c$)
