Foundation Maths for
First Year Physics

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Study Guide and Problem Sheet/Classwork
Lecture 9: Expansions

Learning Outcomes

Jargon
Permutations, combinations, power series.

Notation
${}^{n}\mathrm{P}_{r}$, ${}^{n}\mathrm{C}_{r}$

Concepts
Calculating numbers of arrangements and selections; using ${}^{n}\mathrm{C}_{r}$ to find the coefficients of a binomial expansion; finding the Maclaurin series for a given function; expanding $(1+x)^n$, where n is not a positive integer; restrictions on values of x for such an expansion; expanding $(a+x)^n$.

Problems

1. Calculate:
   (a) ${}^{7}\mathrm{P}_{3}$, (b) ${}^{4}\mathrm{P}_{4}$, (c) ${}^{7}\mathrm{P}_{1}$,
   (d) the number of different 3 digit, odd numbers which can be formed from the set 1, 2, 3, 4, 5, assuming that no member of the set is used more than once.

2. Calculate:
   (a) ${}^{7}\mathrm{C}_{3}$, (b) ${}^{4}\mathrm{C}_{4}$, (c) ${}^{7}\mathrm{C}_{1}$,
   (d) the number of ways in which a committee of five can be selected from a group of 12 people.

3. Find:
   (a) the coefficient of x^5 in $(1+x)^{20}$
   (b) the coefficient of x^3 in $(1+2x)^5$
   (c) the coefficient of x^{22} in $(3+x)^{25}$
   (d) the coefficient of x in $(2-18x)^{10}$
   (e) the coefficient of x in $(1+4x)^{17}(1-3x)^{41}$

4. (a) Write out the first 5 terms in the infinite series for $(1+x)^{-1}$.
   (b) Write out the first 5 terms in the infinite series for $(1-x)^{-1}$.
   (c) Take another look at Problem Sheet 2, Q. 8(d).

5. (a) Find the first 3 non-zero terms in the Maclaurin series for $\sin x$.

   (b) Write down a condition for being able to use the small angle approximation $\sin x \simeq x$, derived in Problem Sheet 4, Q. 6(b).

   (c) Does an angle of 10^o satisfy the condition?

6. (a) Explain why it is impossible to find a Maclaurin series for $\ln x$.

   (b) It *is* possible to find one for $\ln(1+x)$. Obtain the first three non-zero terms, and use them to calculate $\ln(1.05)$ to three significant figures.

   (c) In Problem Sheet 7, Q. 8(b) we also estimated the value of $\ln(1.05)$. How are the two methods related?

7. For the following binomial expressions, write down the range of values of x for which they can be expanded, and obtain the first three terms of the expansions:

   (a) $(1+x)^{1/2}$ (b) $(1-2x)^{1/10}$ (c) $\dfrac{1}{2+3x}$ (d) $(a+x)^{-2}$

   (e) $(1+2x^2)^{1/3}$

8. Decide if the following statements are true or false:

   (a) the equation $x^2 - x + 1 = 0$ has a repeated root

   (b) $810^o = \dfrac{9\pi}{2}$ radians

   (c) $\cos^{-1}(0) = \sin^{-1}(\pi/2)$

   (d) For $y = x\mathrm{e}^{2x}$ the value of $\dfrac{d^2y}{dx^2}$ at $x = 0$ is 4

   (e) The coefficient of x in the expansion of $(\alpha + \beta x)^{-n}$ is $n\beta/\alpha^{n+1}$