Foundation Maths for First Year Physics

M. Coppins
20.10.04

Study Guide and Problem Sheet/Classwork
Lecture 8: Integration

Learning Outcomes

Jargon

Integrate, integrand, integration constant, indefinite integral, definite integral, upper and lower limits of integration.

Notation

$$\int f(x)dx, \int_a^b f(x)dx$$

Concepts

The connection between differentiation and integration; the difference between indefinite integrals and definite integrals; the need for an integration constant; be able to carry out simple integrals using a change of variable; know how to integrate by parts; know the effect of reversing the limits in a definite integral; know how to use integration to find the area under a curve.

Problems

1. Compile your own table of standard integrals by writing down the indefinite integrals of the following functions (i.e., look back to Lecture 7 and find the functions which, when differentiated, produce these functions):

$$\alpha x^n \quad (n \neq -1) \ ; \quad (\alpha x + \beta)^n \quad (n \neq -1) \ ; \quad \frac{1}{x} \ ; \quad \frac{1}{\alpha x + \beta} \ ; \quad \mathrm{e}^{\gamma x} \ ; \quad \cos x \ ; \quad \sin x \ .$$

2. Find the following indefinite integrals

(a) $\int x^5 dx$ (b) $\int x^{-5} dx$ (c) $\int 8(4x+1)^{-1/3} dx$ (d) $\int \frac{6x^2}{(2x^3+4)^2} dx$

(e) $\int \cos(\omega t) dt$

3. Use integration by parts to find the following indefinite integrals:

(a) $\int x\mathrm{e}^{-x} dx$ (b) $\int x \cos x dx$ (c) $\int x^2 \ln x dx$

4. Sometimes an integration by parts is (even) less straightforward. Possibilities include:

(i) the need to repeat the process for the integral on the right hand side, and,

(ii) regarding the integral as multiplied by 1 and putting $\frac{du}{dx} = 1$.

Use integration by parts to find the following indefinite integrals

(a) $\int x^2 \cos x dx$ (b) $\int \ln x dx$ (c) $\int \mathrm{e}^x \sin x dx$

5. Evaluate the following definite integrals:

(a) $\displaystyle\int_0^1 x^5 dx$ (b) $\displaystyle\int_{-1}^1 \frac{6x^2}{(2x^3+4)^2} dx$ (c) $\displaystyle\int_0^{\pi/4} \cos(2t)dt$ (d) $\displaystyle\int_0^2 x\mathrm{e}^{-x} dx$

(e) $\displaystyle\int_1^2 \ln x dx$

6. Find the area of each of the regions bounded by the lines or curves specified:

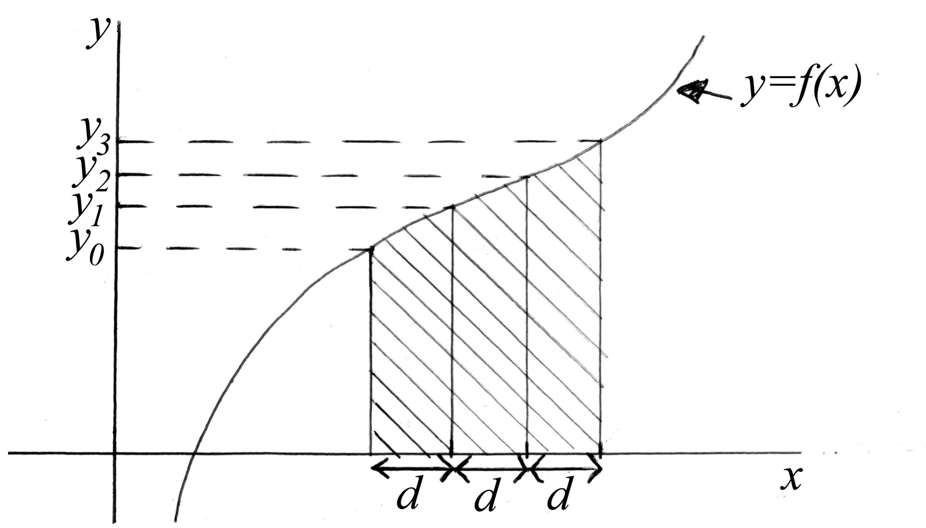
(a) the x axis, the curve $y = x^2$, the line $x = 1$, and the line $x = 2$.

(b) the line $x = 2$, the line $x = 5$, the curve $y = 2\mathrm{e}^x$, and the x axis.

(c) the curve $y = x^2 + 3$, the line $y = x$, the line $x = 0$, and the line $x = 2$.

7. In Problem Sheet 4 we obtained the following identity: $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$. Use this to write down an expression for $\cos^2 x$ in terms of $\cos(2x)$, and, hence, evaluate $\displaystyle\int_0^{2\pi} \cos^2 x dx$.

8. (a) The figure shows an area under the curve $y = f(x)$ divided into three vertical strips of equal width, d. The vertical edges of the strips have heights y_0, y_1, y_2 and y_3, as shown. By approximating the area of each strip by the area of a trapezium, show that the total area is approximately $\frac{d}{2}(y_0 + 2y_1 + 2y_2 + y_3)$.



(b) Write down the corresponding expression for five strips.

(c) Use the five strip formula to find an approximate value of $\displaystyle\int_1^2 x^2 dx$. Compare it with the correct value, obtained in Q. 6(a).