Foundation Maths for First Year Physics — M. Coppins — 18.10.04

Study Guide and Problem Sheet/Classwork
Lecture 6: e

Learning Outcomes

Jargon

Exponential growth, exponential decay, e-folding time, half-life.

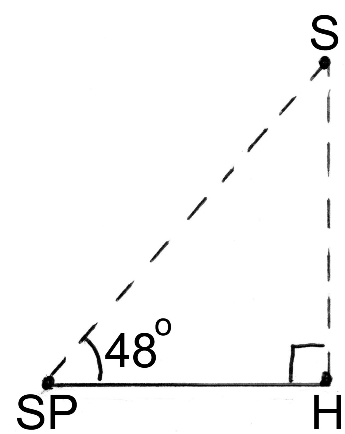
Concepts

How to find the gradient at a point on a curve geometrically.

Problems

As physicists we don't just need to know a whole lot of maths, we need to be able to apply the maths to solve 'real' problems. In contrast to the previous problem sheets, this one aims to give you some practice in doing this. It involves only maths covered in the lectures so far.

1. Ordnance Survey *trig points* are concrete or stone pillars, about 1.3 m high, dotted around the country on hilltops. 'Trig' is short for trigonometric. The diagram shows schematically the relative positions of the trig points on three mountains in the English Lake District: Scafell Pike (SP), Helvellyn (H), and Skiddaw (S). The position of Skiddaw can be determined from measurements made on the other two mountains, which reveal the angles shown in the diagram, while it is known that Scafell Pike and Helvellyn are separated by a distance of 14.7 km. What are the distances between Scafell Pike and Skiddaw, and between Helvellyn and Skiddaw?



2. A water tank has a square base, of side 2 m, and vertical sides. The depth of water in the tank is initially 0.5 m. At some time a tap is opened and water starts to flow into the tank at a constant rate. 10 minutes later the depth of water is 2.0 m.

   (a) Sketch a graph of V (= volume of water in the tank, in m^3) as a function of t (= time from opening the tap, in s).

   (b) Find the rate of flow (in m^3/s) of water into the tank.

   (c) Write down an equation for V as a function of t.

3. I travel up to London from Brighton by train, a distance of 80 km. Sometimes the train has to go slower because of flash floods and landslides in the South Downs (this happens surprisingly often, actually). On one such occasion the average speed of the train was reduced by 20 km/hr from its usual value, and, as a result, the journey took 30 minutes longer than usual. How long does the journey usually take?

4. Boltzmann's Bakery are considering closing down their Special Order Section. The luxury cakes made by the section are sold at an average price of £50. The cost (in pounds) of making n cakes per week is $20\sqrt{n}$ (it has this form because, for large numbers the ingredients can be bought in bulk; note that $\sqrt{n}$ must be positive). In addition, the section has running costs of £5000/week, which would be incurred even if no cakes were ordered.

   (a) Sketch on the same set of axes the weekly income (from the sale of cakes) and weekly expenditure (cost of making them plus running costs) as a function of n (the number cakes ordered per week).

   (b) What is the minimum number of orders per week if the section is to be profitable?

5. An alternating current varies with time according to $I(t) = I_0 \sin(\omega t)$, where t is in seconds, and ωt is in radians.

   (a) What is the unit of ω?

   (b) What is $I(t = 0)$?

   (c) At what time (in terms of ω) is I next equal to $I(t = 0)$?

   (d) When is it next equal to $I(t = 0)$ after that?

   (e) Write down a general formula (in terms of ω) for the times at which $I = I(t = 0)$.

6. The population of a human colony on some distant planet is given by $P = 10^3\,\mathrm{e}^{0.01t}$ where t is the time in years from the initial settlement.

   (a) How many settlers landed on the planet initially?

   (b) How many years after the initial settlement does the population reach 1 million?

   (c) How many more years does it take for the population to reach 2 million?

7. A quantity whuch decreases with time proportional to $\mathrm{e}^{-\gamma t}$ (where $\gamma > 0$) is said to be undergoing *exponential decay*. The mass of a radioactive substance has this property.

   A piece of radioactive material has a mass, in kg, given by $M = 2.0\,\mathrm{e}^{-0.001t}$, where t is the time elapsed, in seconds, from the initial measurement of the mass

   (a) What was the initial mass?

   (b) Sketch a graph of M against t.

   (c) What is the mass after 20 minutes?

   (d) What is the *half-life* (i.e., the time it takes for the mass to halve)?

8. (a) In 1965 Gordon Moore, one of the co-founders of Intel, proposed the following 'law': the number of transistors which can be fitted onto an integrated circuit doubles every 18 months. Remembering that any quantity growing exponentially, proportional to $\mathrm{e}^{\gamma t}$ (t in years), doubles every $\frac{1}{\gamma}\ln 2$ years, we see that Moore's law can be written as $n =$ number of transistors/integrated circuit $= n_0\mathrm{e}^{\gamma t}$. What is the value of γ (in years^{-1})?

   (b) When Moore proposed his law, in 1965, integrated circuits carried 64 transistors. In 2000 Intel introduced the Pentium 4 which has 4.2×10^7. Assuming exponential growth, what value of γ would be consistent with these numbers? How close was Moore's estimate of 18 months for the doubling period?